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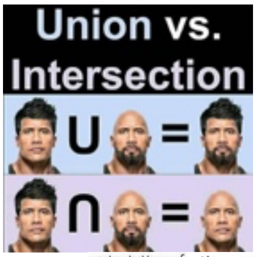
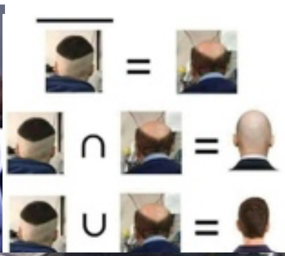
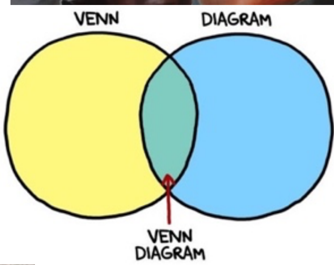
Probability: Venn Diagrams







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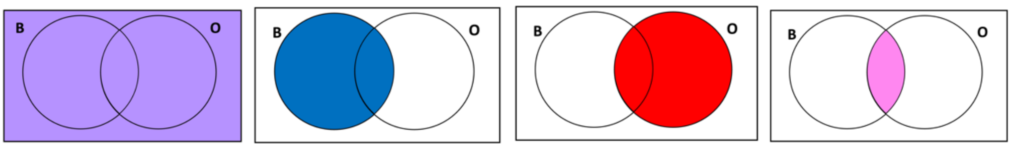
1)

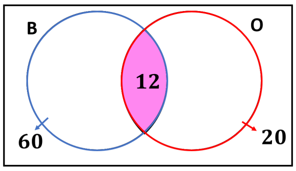
100 students total
60 are in the band
20 are in the orchestra.
12 students are in both the band and the orchestra

Let's show what these 4 sentences represent these visually on a diagram

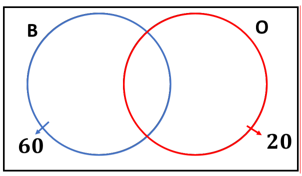


Let's put this on one diagram. We always start with filling in the central part (the overlap). This is because after this we can easily work outwards to know the number in each region

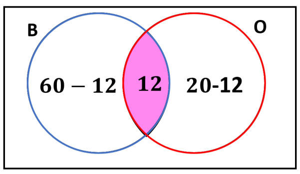
12 students are in both band and the orchestra means that the central overlap represents 12



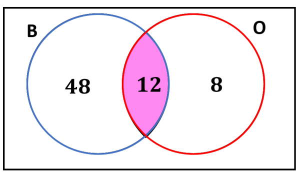
We are also given that 60 are in the band (entire left circle) and 20 are in the orchestra (entire right circle)



Hence



Simplify



Now, the piece of information: "The Pythagoras School of Music has 100 students", we know that the entirety of the Venn diagram should add up to 100.

This means we can easily work out the outside region

Way 1: Without Algebra

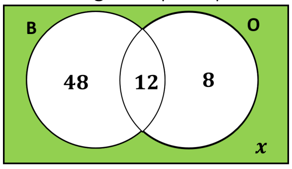
Take the numbers away from the total of 100

$$100 - 48 - 12 - 8$$

$$= 32$$

Way 2: Algebra (Build an equation)

Assuming that the area outside of both sets is x, we can find its value using a simple equation.



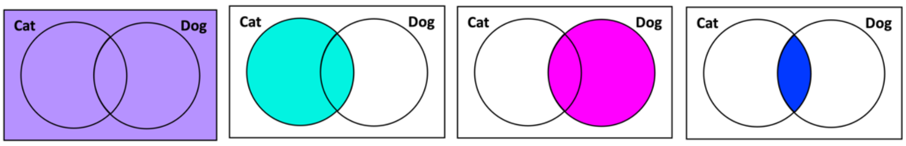
We know the total is 100

$$48 + 12 + 8 + x = 100$$
$$100 = 68 + x$$
$$x = 100 - 68$$
$$x = 32$$

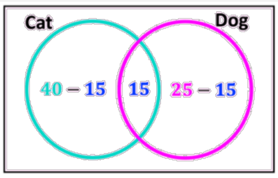
Therefore, there are 32 students that are in neither the band nor the orchestra.

2)

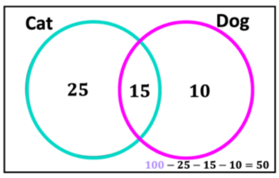
100 total
40 own a cat.
25 own a dog
15 own a cat and a dog



Fill in the centre part (overlap) first and then we can easily work out the other regions (left and right sections)



Let's simplify the numbers and fill in the outside part



i. $\frac{25+15+10}{100} = \frac{50}{100} = \frac{1}{2}$

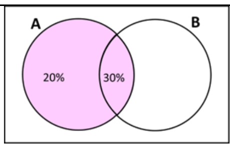
ii. $\frac{25+10}{100} = \frac{35}{100} = \frac{7}{20}$

iii. $\frac{15}{25+15} = \frac{15}{40} = \frac{3}{8}$

iv. $\frac{10}{15+10} = \frac{10}{25} = \frac{2}{5}$

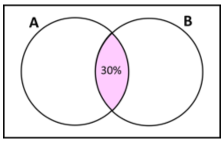
3)

i.



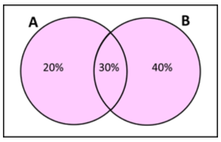
20% + 30% = 50% = 0.50

ii.



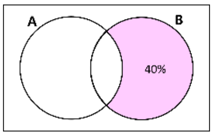
30% = 0.30

iii.



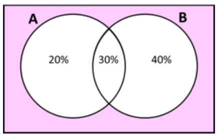
20% + 30% + 40% = 90% = 0.90

iv.



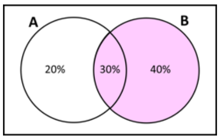
40% = 0.40

v.



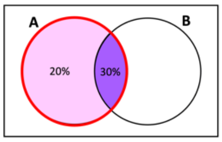
100% - 20% - 30% - 40% = 100% - 90% = 10% = 0.10

vi.



30% + 40% = 70% = 0.70

vii.



$$\frac{30\%}{20\% + 30\%} = \frac{30\%}{50\%} = \frac{3}{5} = 0.6$$

4)

Smokes and gets cancer = 0.05

Smoke and does not get cancer = 0.20

Does not smoke and gets cancer= 0.03

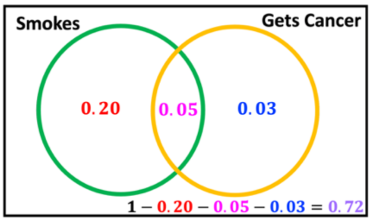
Does not smoke and does not get cancer

The sum of all probabilities = 1

0.05 + 0.20 + 0.03 + does not smoke and does not get cancer = 1

Re-arrange for does not smoke and does not get cancer

does not smoke and does not get cancer = 1 – 0.20 – 0.05 – 0.03 = 0.72



i. $\frac{\text{gets cancer and is smoker}}{\text{total smokers}} = \frac{0.05}{0.2+0.05} = \frac{0.05}{0.25} = \frac{1}{5} = 0.2$

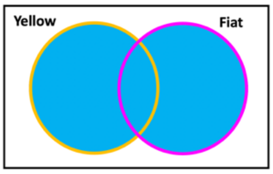
ii. $\frac{\text{does not get cancer and is a smoker}}{\text{total smokers}} = \frac{0.2}{0.2+0.05} = \frac{0.2}{0.25} = \frac{4}{5} = 0.8$

iii. $\frac{\text{non-smokers who get cancer}}{\text{total non-smokers}} = \frac{0.03}{0.03+0.72} = \frac{0.03}{0.75} = 0.04$

iv. $\frac{\text{does not get cancer and is non-smoker}}{\text{total non-smokers}} = \frac{0.72}{0.03+0.72} = \frac{0.72}{0.75} = 0.96$

5)

P(yellow or a fiat) = 0.6



P(yellow)= 0.4

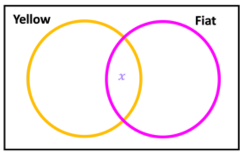
P(fiat)= 0.3

It looks like there is not enough information to fill anything in. To get started we always want the centre part first

Way 1: Use Algebra

(avoids you having to think)

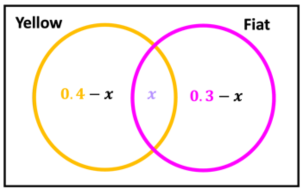
We don't know the middle part, lets call it x



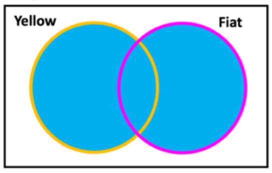
Let's now use the fact that

P(yellow)= 0.4

P(fiat)= 0.3



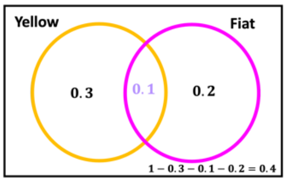
We can form an equation based on knowing the total of the circles is 0.6



$$0.4 - x + x + 0.3 - x = 0.6$$

$$0.7 - x = 0.6$$

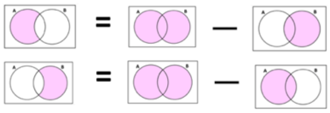
$$x = 0.1$$



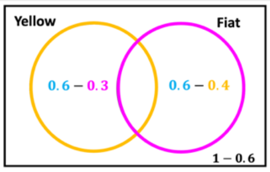
Probability of **not a** yellow fiat means not the purple part (not the intersection)

$$0.3 + 0.2 + 0.4 = 0.9$$

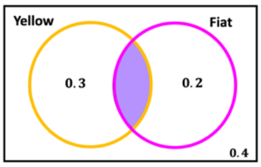
Way 2: Use symmetry of the Venn diagram



We use the top result to fill in the left crescent circle and the bottom results to fill in the right crescent circle below



Simplify



To find what the purple shaded region use the fact that inside the circles is 0.6

0.3 + 0.2 + both = 0.6

both $= 0.1$

Note: or we could have used the sum of all probabilities = 1

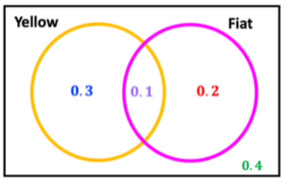
0.3 + 0.2 + 0.4 + both = 1

Re-arranging to find both

both $= 1 - 0.3 - 0.2 - 0.4$

both $= 0.1$

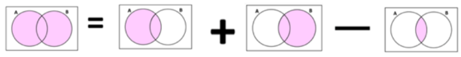
We can fill this into the Venn diagram



Probability of **not a** yellow fiat means not the purple part (not the intersection)

$$0.3 + 0.2 + 0.4 = 0.9$$

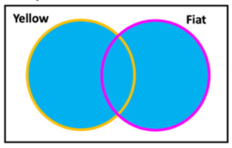
Way 3: Use symmetry of the Venn diagram



This rule should make sense. When we add A and B we add the middle section twice (double count it) so we must take it away after.

We use the rule to work out the central part (this rule is known as the addition formula).

Tailoring this to our question



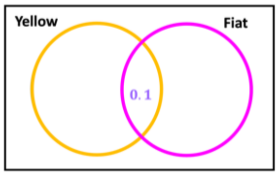
P(yellow or a fiat) = P(yellow)+P(fiat) − P(yellow & fiat)

-

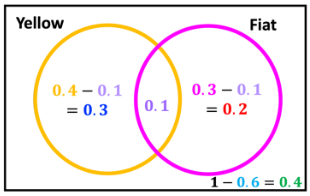
0.6 = 0.4+0.3 − P(yellow and fiat)

P(yellow and fiat)= 0.1

So, we can fill in the middle section first



Now we can fill in the rest

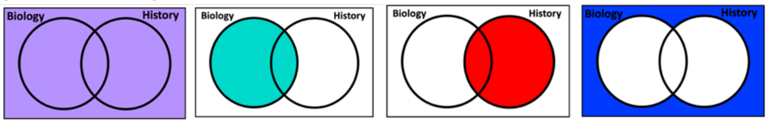


Probability of **not a** yellow fiat means not the purple part (not the intersection)

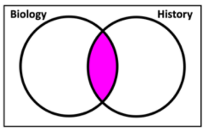
$$0.3 + 0.2 + 0.4 = 0.9$$

6)

Total 20 students
12 study biology
15 student history
2 students study neither Biology nor History
These are represented by the following on a Venn diagram

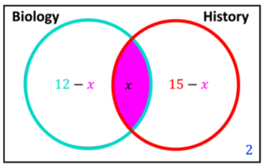


This is harder since we can't fill in each of the 3 separate parts inside the circles. It looks like there is not enough information to fill anything in.
To get started we always want the centre part first



Way 1: With algebra (avoids you having to think)

We don't know the middle part, lets call it x



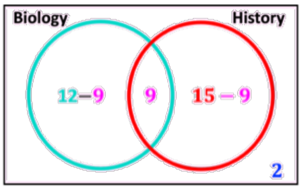
We can form an equation based on knowing total of the entire diagram is 20

$$12 - x + x + 15 - x + 2 = 20$$

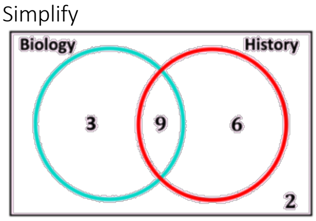
$$29 - x = 20$$

$$x = 9$$

Put x found back



Simplify



$\frac{9}{20}$

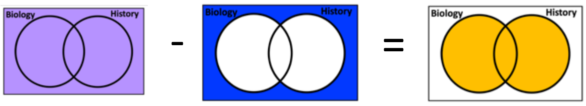
$\frac{9}{3+9} = \frac{9}{12} = \frac{3}{4}$

$$\frac{2}{20} = \frac{1}{10}$$

$$\frac{3+6+2}{20} = \frac{11}{20}$$

Way 2: Without algebra (need to think a bit more)

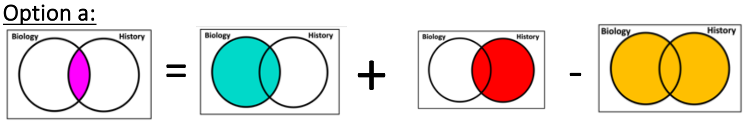
We can work the total of inside the circles first by using



$20 - 2 = 18$ study one of the subjects

Now that we know there orange there are 2 options to proceed from here to find the centre part:

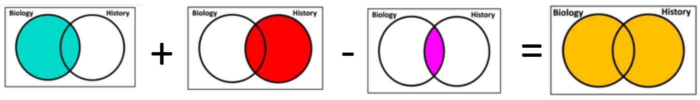
<u>Option a:</u>



This should make sense since by adding the green and red we double count the pink centre and by removing everything we are simplify le with the link centre

$$\text{both subjects} = 12 + 15 - 18$$
$$\text{both subjects} = 9$$

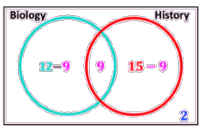
Another way to think of this is



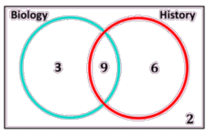
This should make sense since by adding the green and red we double count the centre and have to remove it

$$12 + 15 - \text{both subjects} = 18$$
$$\text{both subjects} = 9$$

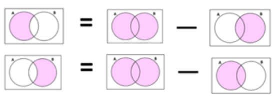
Recall we have12 study biology and15 student history. So, we are left with

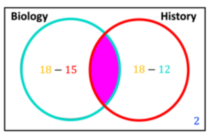


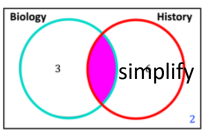
simplify



<u>Option b:</u> Consider the following template







To find what the pink shaded region use the fact that inside the circles is 18
3 + 6 + both = 18
both = 9
Note: or we could use the fact that the total is 20
3 + 6 + both+2 = 20

both $= 9$

Let's fill this in

$$\frac{9}{20}$$

$$\frac{9}{3+9} = \frac{9}{12} = \frac{3}{4}$$

$$\frac{2}{20} = \frac{1}{10}$$

$$\frac{3+6+2}{20} = \frac{11}{20}$$

7)

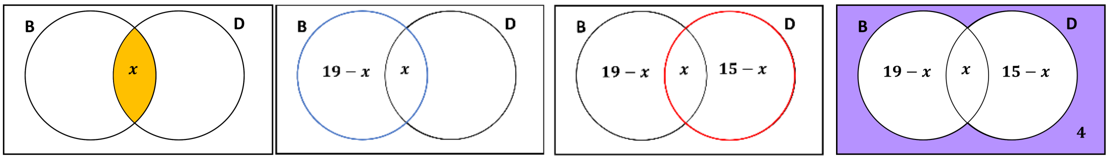
Assume x adults booked breakfast and dinner

19 adults booked breakfast

15 adults booked dinner

4 adults did not book breakfast or dinner

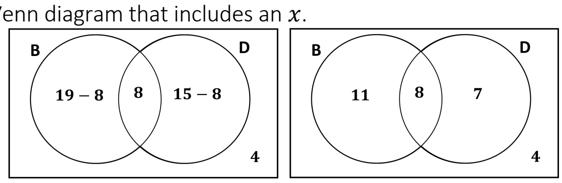
30 adults total



Knowing that the total number of adults that were asked was 30, we know that all of the values on the Venn diagram should add up to 30. With this information, we can make a simple equation to find the value of x.

$$(19 - x) + x + (15 - x) + 4 = 30$$
$$30 = 38 - x$$
$$x = 38 - 30$$
$$x = 8$$

Let's now fill in and simplify everything on the Venn diagram that includes an x.



The denominator of a probability fraction is always the number of the sample of which something is being chosen from. From the question, we know that there are 30 adults. This means that 30 is our denominator.

The numerator is number that booked breakfast **and** dinner which is 8 as we can see from the diagrams in the area where both sets overlap. So, the probability that a random adult chosen booked both breakfast and dinner is

$$\frac{8}{30}$$

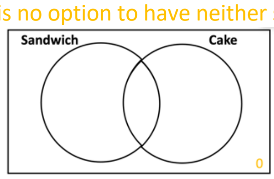
However, the question asked the probability that **two** adults chosen booked both breakfast and dinner. This means we have to take the probability that one adult chosen booked both breakfast and dinner then subtract 1 from the values as one would have been removed after being chosen. Then we have and multiply them by each other like so:

$$\frac{8}{30} \times \frac{7}{29} = \frac{28}{435}$$

8)

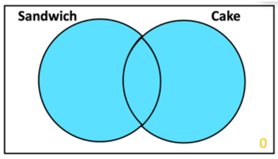
We know all probabilities must add to one.

The customer must either have a sandwich or a cake (i.e. there is no option to have neither so the outside is empty)



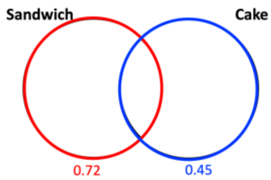
So, the inside of the Venn diagram must add to 1. This means

P (sandwich or cake)= 1

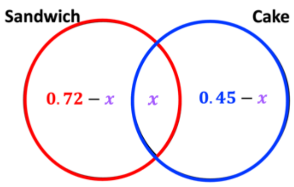


P(sandwich)= 0.72

P (cake)= 0.45



Way 1: Algebra



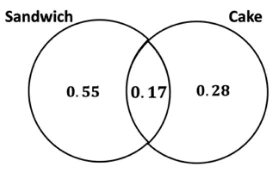
Total is 1

$$0.72 - x + x + 0.45 - x = 1$$

$$1.17 - x = 1$$

$$x = 0.17$$

Put x found back



Way 2: Probabilities inside add to 1

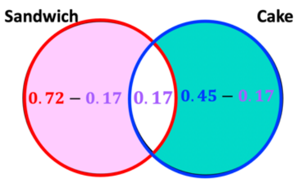
only a sandwich + only a cake + both=1

1 = (sandwich – both) + (cake – both) + both

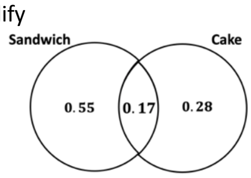
$1 = \text{sandwich} + \text{cake} - \text{both}$

$1 = 0.72 + 0.45 - \text{both}$

$\text{both} = 0.17$



Simplify



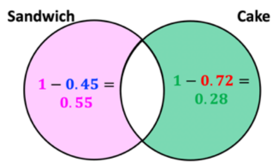
Way 3: only sandwhich + cake =1

$\text{Only a sandwich} = 1 - \text{cake}$

$\text{Only a sandwich} = 1 - 0.45$
$= 0.55$

$\text{Only a cake} = 1 - \text{sandwich}$

$\text{Only a cake} = 1 - 0.72 = 0.28$

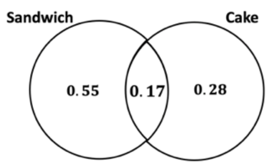


We can find the inside part now since probabilities add to 1

$0.55 + 0.28 + \text{both} = 1$

$\text{both} = 1 - 0.55 - 0.28 = 0.17$

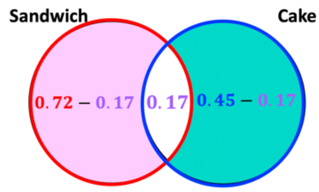
So, we have



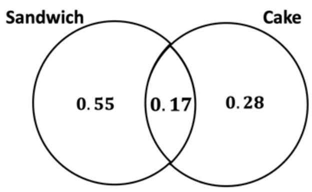
Way 4: Addition Formula

P(sandwich or cake)
=P(sandwich)+p(cake)−P(sandwich & cake)

1 =0.72+0.45−P(sandwich and cake)



Simplify



We can easily read off the answers from the Venn diagram now

i. 0.17
ii. 0.55
iii. probability each customer buys a cake = $0.17 + 0.28 = 0.45$
200 expected customers a day
So expected number of cakes = $0.45 \times 200 = 90$

9)

To begin, let’s classify each of the given numbers as rational numbers, integers, or both, or neither. Before we do, let’s define both rational numbers and integers.

A rational number is a number that can be expressed as the quotient or fraction of two integers, where the denominator is not zero.

An integer is the number zero (0), a positive natural number (1, 2, 3, . . .), or the negation of a positive natural number (−1, −2, −3, . . .).

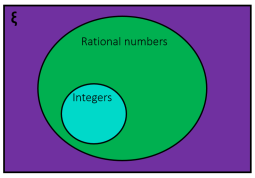
Firstly, 2.6. It is considered a rational number as it can be expressed as $\frac{13}{5}$ but is not an integer as it is not a whole number.

Secondly, $\frac{4}{17}$. It is considered a rational number, but is not an integer

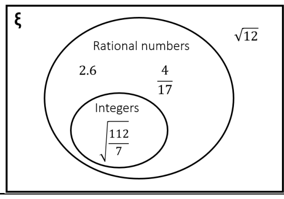
Thirdly, $\sqrt{12}$. It isn’t considered a rational number nor an integer.

Lastly, $\sqrt{\frac{112}{7}}$. It is considered a rational number and an integer because, when simplified, it equals 4.

Now let’s fill in the diagram, numbers that are considered integers and rational numbers are going into the area shaded turquoise. Numbers that are considered rational numbers but not integers are going to the area shaded green. Numbers that are neither rational nor integers go in the area shaded purple.



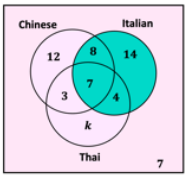
Now fill in the numbers a stated earlier.



1.2 3 events

10)

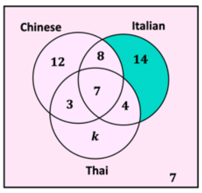
i.



Probability they like Italian

$$\frac{8+14+7+4}{60}=\frac{33}{60}=\frac{11}{20}$$

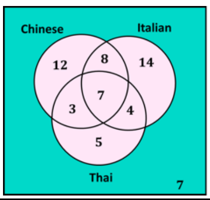
ii.



Probability they like only Italian

$$\frac{14}{60}=\frac{7}{30}$$

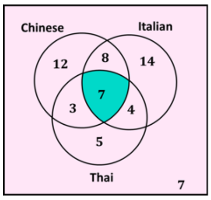
iii.



Probability they like none of these choices

$$\frac{7}{60}$$

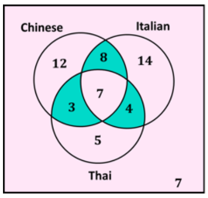
iv.



Probability they like all of these choices

$$\frac{7}{60}$$

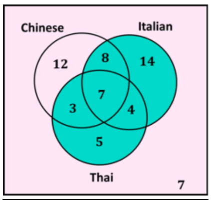
v.



Probability they like only two out of three

$$\frac{8+3+4}{60}=\frac{15}{60}=\frac{1}{4}$$

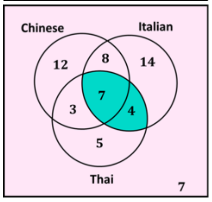
vi.



Probability they like Thai or Italian

$$\frac{8+14+7+3+4+5}{60}=\frac{41}{60}$$

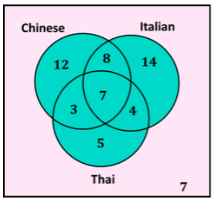
vii.



Probability they like Italian and Thai

$$\frac{7+4}{60}=\frac{11}{60}$$

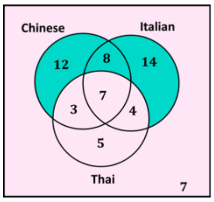
viii.



Probability they like at least one of these choices

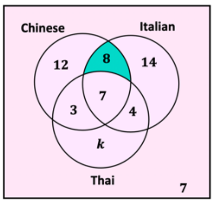
$$\frac{12+8+14+7+3+4+5}{60}=\frac{53}{60}$$

ix.



Probability they like Chinese or Italian, but not Thai

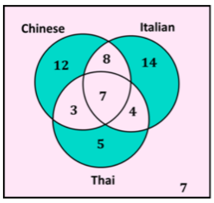
$$\frac{12+8+14}{60}=\frac{34}{60}$$



Probability they like Chinese and Italian, but not Thai

$$\frac{8}{60}=\frac{2}{15}$$

x.

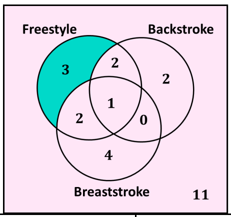


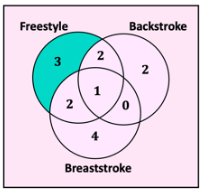
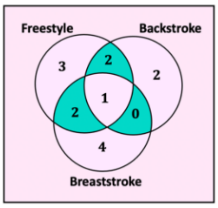
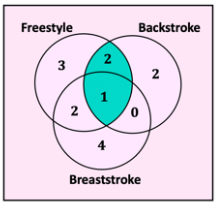
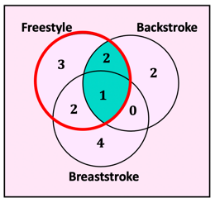
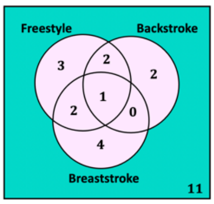
Probability they like exactly one of the three

$$\frac{12+5+14}{60}=\frac{31}{60}$$

11)

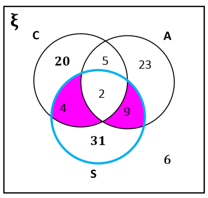
Swimmers who swim none of these events = $25-3-2-2-2-1-0-4=25-14=11$



i.		P(swims only freestyle) = $\frac{3}{25}$
ii.		P(swims exactly 2 events) = $\frac{2+2+0}{25}=\frac{4}{25}$
iii.		P(swims freestyle and backstroke) = $\frac{2+1}{25}=\frac{3}{25}$
v.		P(swims backstroke given swims freestyle) $=\frac{\text{swims backstroke and freestyle}}{\text{swims freestyle}}=\frac{2+1}{3+2+1+2}=\frac{3}{8}$
vi.		P(does not swim freestyle, backstroke or breaststroke) = $\frac{11}{25}$

12)

The denominator of a probability fraction is the number of people that are being chosen from, in this case, it will be the people who like science fiction which are $31+4+2+9=46$ people.
The numerator of this fraction will be the number of people after the criteria has been satisfied. In this case, people who also liked **one** other movie which is shaded below.



As we can observe from the diagram, $4+9=13$ people like science fiction and only one other movie. Making 13 our numerator. Which makes our probability fraction as so:

$$\frac{13}{46}$$

2 Silver

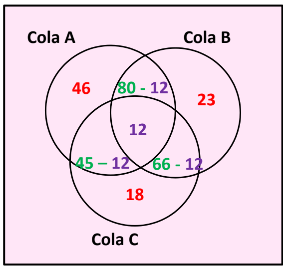
2.1 3 events

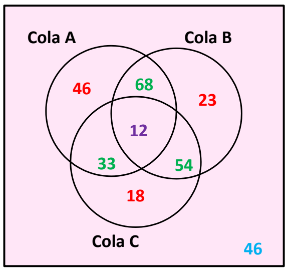
13)

46 like only Cola A, 23 like only Cola B, 18 like only Cola C

80 like both Colas A and B, 66 like both Colas A and C, 45 like both Colas B and C

12 like all three Colas





Note: 300 in total, so we can find the outside region

$300 - 46 - 68 - 23 - 12 - 33 - 54 - 18 = 46$ don't like any Cola

i. Likes all three colas $= \frac{12}{300} = \frac{1}{25}$

ii. Likes colas A and B $= \frac{68+12}{300} = \frac{80}{300} = \frac{4}{15}$

iii. Likes Cola A or Cola B$= \frac{46+68+23+12+33+54}{300} = \frac{236}{300} = \frac{59}{75}$

iv. Does not like Cola A or B or C$= \frac{46}{300} = \frac{23}{150}$

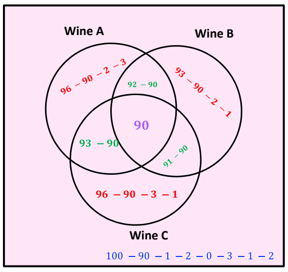
v. Likes at least 2 Colas $= \frac{68+12+33+54}{300} = \frac{167}{300}$

vi. Likes only 1 Cola $= \frac{46+23+18}{300} = \frac{87}{300} = \frac{29}{100}$

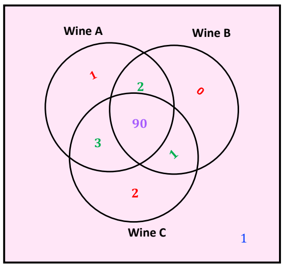
vii. Likes B and does not like C $= \frac{68+23}{300} = \frac{91}{300}$

14)

90 like all 3
92 like A and B, 91 like B and C, 93 like A and C
96 like A, 93 like B, 96 like C
100 in total



Simplify



i. $\frac{1}{100}$

ii. $\frac{1+3}{100}=\frac{4}{100}$

iii. $\frac{1+2+0}{100}=\frac{3}{100}$

iv. $\frac{1+2+0}{100}=\frac{3}{100}$

v. $\frac{2+3+1}{100}=\frac{6}{100}$

vi. $\frac{1+2+0+90+3+1+2}{100}=\frac{99}{100}$

vii. $\frac{1}{100}$

viii. $\frac{2}{100}$

ix. $\frac{1+2+90+3+1}{100}=\frac{97}{100}$

x. $\frac{1+2+0}{100}=\frac{3}{100}$

xi. $\frac{\text{like A and C}}{\text{total that like A}}=\frac{3+90}{90+1+2+3}=\frac{93}{96}$

xii. $\frac{\text{likes A or B or both AND doesn't like A}}{\text{likes A or B or both}}=\frac{\text{likes B and doesn't like A}}{\text{likes A or B or both}}=\frac{0+1}{1+2+0+90+3+1}=\frac{1}{97}$

xiii. A and B are independent if the probability that a person likes A multiplied by the probability a person likes B equals the probability that a person likes A and B.

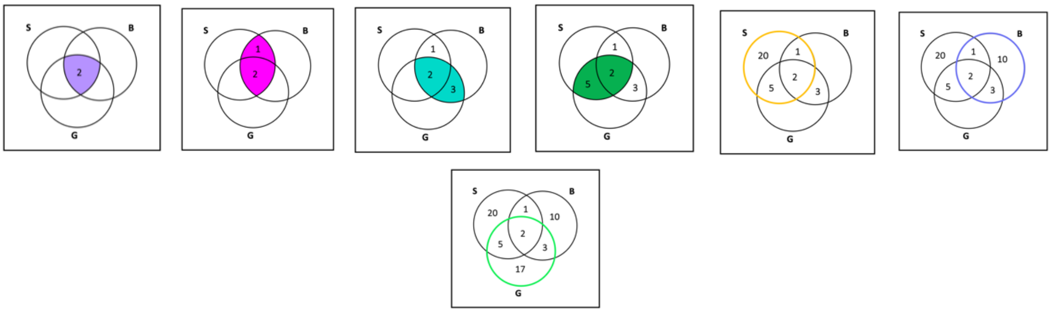
- probability a person likes A $=\frac{90+1+2+3}{100}=\frac{96}{100}$
- probability a person likes B $=\frac{90+0+2+1}{100}=\frac{93}{100}$
- probability a person likes A and B $=\frac{90+2}{100}=\frac{92}{100}$

$$\frac{96}{100}\times\frac{93}{100}\neq\frac{92}{100}$$

15)

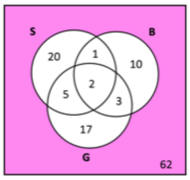
a) Let’s start by drawing the Venn diagram and filling in the first three pieces of information
We always fill in the intersection in the very middle and then we can and work from there

2 people went swimming, played basketball, and used the gym
3 people went swimming and played basketball
5 people played basketball and used the gym
7 people went swimming and used the gym
28 people went swimming
16 people played basketball
27 people used the gym

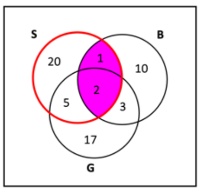


Finally, subtract all of the numbers that are shown in the diagram from 120 to find the amount of people that don’t swim, nor play basketball, nor go to the gym and are going to be outside all of the sets.

$$120-(20+1+5+2+3+10+17)=62$$



b) We are restricted to the people swimming since we are told one of the people who went swimming is chosen



$$\frac{3}{28}$$

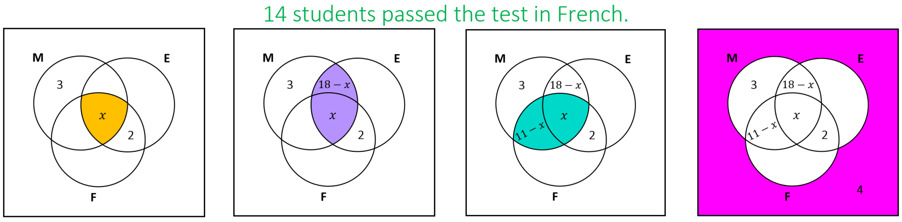
3 Gold

3.1 With Algebra

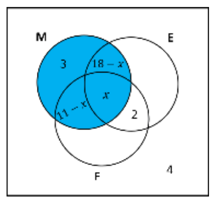
16)

a)

x students passed all three tests.
18 students passed the tests in Maths and English.
11 students passed the test in Maths and French.
4 students failed all three tests.
25 students passed the test in Maths.
20 students passed the test in English.
14 students passed the test in French.

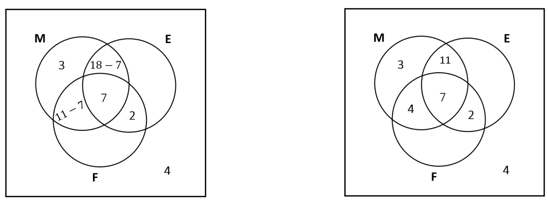


"25 students passed the test in Maths." We can make a simple equation to find the value of x since all of the values of set M is known in terms of x

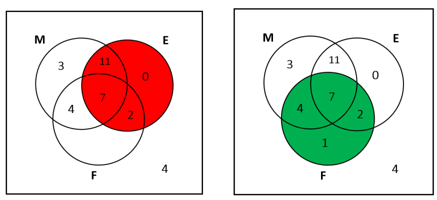


$$25 = 3 + (18 - x) + x + (11 - x)$$
$$25 = 32 - x$$
$$x = 32 - 25$$
$$x = 7$$

b) Firstly, fill in x and simplify:

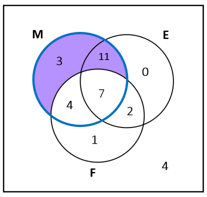


Then, fill in the final missing spaces using the last two pieces of information.



c) The denominator of a probability fraction is always the number of the sample of which something is being chosen from. In this case, it is the students who passed the test in Maths. And from the question, we know that 25 students passed the test in maths. This means that 25 is our denominator.

The numerator will be the number of students left after a certain criteria is fulfilled. In this case, it is the students who passed the test in Maths but failed the test in French. Let's start by shading the area on the Venn diagram of those who passed the test in Maths. That will be everything in set M, but unshade everything that overlaps with set F as our criteria calls for those who also failed French.



As we can see from the diagram, the number of students left is $3 + 11 = 14$ students. This means that our numerator is equal to 14 and our probability fraction is as so:

$$\frac{14}{25}$$

17)

a)

4 study all three of Russian, French, and German

10 study Russian and French

13 study French and German

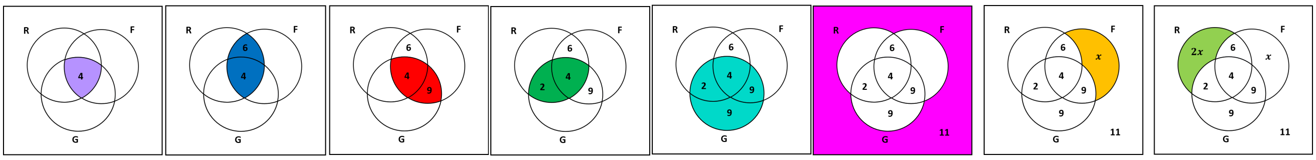
6 study Russian and German

24 study German

11 study none of the three subjects

Let x be the number of students who study French only.

The number who study Russian only is twice the number who study French only. $(2x)$



b) "The number of students who were asked the question was 80". This tells us that the total value of the Venn diagram when added up should equal to 80. By making an equation, we can find the value of x like so:

$$80 = 2x + x + 6 + 2 + 4 + 9 + 9 + 11$$
$$80 = 3x + 41$$
$$3x = 80 - 41$$
$$3x = 39$$
$$x = 13$$

Knowing that the number of students who study Russian only is $2x$, we can substitute x for 13 and find the number of students who studied Russian only.

$$2x = 2(13)$$
$$= 26 \text{ students}$$

Knowing this, we can tell that the total number of students who studied Russian is:

$$36 + 2 + 4 + 6 = 36 \text{ students}$$

18)

a)

6 have a dog, a cat, and a rabbit

x have both a dog and a cat

8 have both a cat and a rabbit

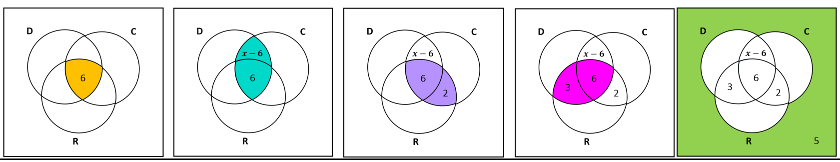
9 have both a dog and a rabbit

5 have not got a dog, a cat, or a rabbit

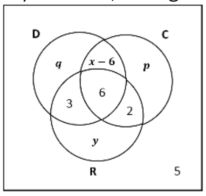
28 have a dog

18 have a cat

20 have a rabbit



Now, we can fill in the last of the empty spaces. For clarity reasons, let's give a symbol for each of the empty spaces.



Let's start by finding the value of y. Since we know that the entirety of set R is equal to 20 so we can make a simple equation to find its value.

$$20 = 3 + 6 + 2 + y$$
$$20 = 11 + y$$
$$y = 20 - 11$$
$$y = 9$$

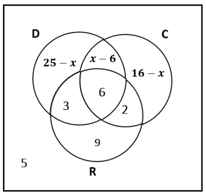
Next, we have the same process for q with set D. We just have to be mindful of the unknown value x.

$$28 = 3 + 6 + (x - 6) + q$$
$$28 = 3 + x + q$$
$$q = 28 - 3 - x$$
$$q = 25 - x$$

Finally, p with set C.

$$18 = 2 + 6 + (x - 6) + p$$
$$18 = 2 + x + p$$
$$p = 18 - 2 - x$$
$$p = 16 - x$$

Now, just fill in the Venn diagram for our final answer.



b) Now, we know that a total of 50 students participated in the survey. This means that all of the value on the Venn diagram should add up to 50. Knowing this, we can make a simple equation to find the value of x.

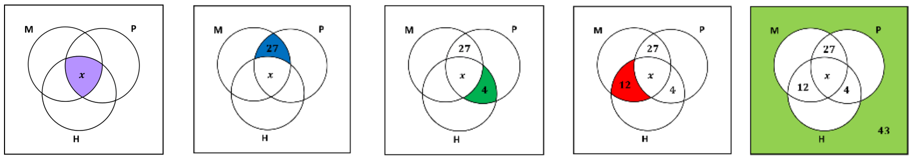
$$50 = (25 - x) + (x - 6) + (16 - x) + 3 + 6 + 2 + 9 + 5$$
$$50 = 60 - x$$
$$x = 60 - 50$$
$$x = 10$$

4 Diamond

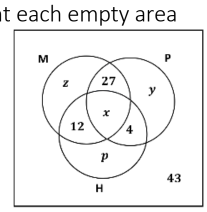
4.1 With Algebra

19)

Assume that x students study all three subjects
27 students are studying Mathematics and Physics, but not History
4 are studying History and Physics, but not Mathematics
12 are studying History and Mathematics, but not Physics.
65 are studying Mathematics
38 are studying Physics
49 are studying History
43 study none of these subjects



Now, for clarity reasons, let's put a symbol to represent each empty area



Let's start by finding the value of z. Since we know that the entirety of set M is equal to 65 so we can make a simple equation to find its value.

$$65 = z + 27 + x + 12$$
$$65 = z + x + 39$$
$$z = 65 - 39 - x$$
$$z = 26 - x$$

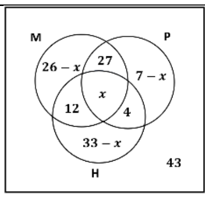
Next, we have the same process for y with set P. We just have to be mindful of the unknown value x.

$$38 = y + x + 27 + 4$$
$$38 = y + x + 31$$
$$y = 38 - 31 - x$$
$$y = 7 - x$$

Finally, p with set H.

$$49 = p + x + 12 + 4$$
$$49 = p + x + 16$$
$$p = 49 - 16 - x$$
$$p = 33 - x$$

Now, just fill in the Venn diagram.



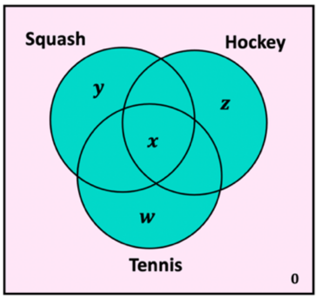
Now, we know that a total of 142 A-level students. This means that all of the value on the Venn diagram should add up to 142. Knowing this, we can make a simple equation to find the value of x.

$$142 = (26 - x) + 27 + (7 - x) + 12 + x + 4 + (33 - x) + 43$$
$$142 = 152 - 2x$$
$$2x = 152 - 142$$
$$2x = 10$$
$$x = 5$$

Therefore, since $x = 5$, we know that there are 5 students who study all three subjects of Maths, Physics, and History.

20)

Way 1



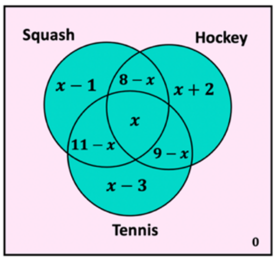
Let's find w, x, y and z in terms of x

We know the following

$$y + 11 + (8 - x) = 18$$
$$y = x - 1$$

$$w + 11 + (9 - x) = 17$$
$$w = x - 3$$

$$z + 8 + (9 - x) = 19$$
$$z = x + 2$$

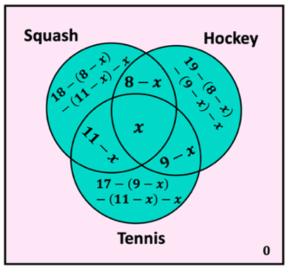


The total of these is equal to 30

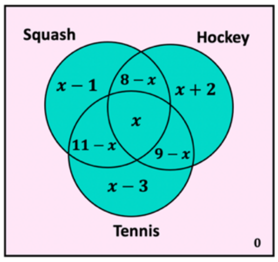
$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$
$$x + 26 = 30$$
$$x = 4$$

Hence the number of people who play all three sports is 4

Way 2



Let's simplify each part by collecting like terms

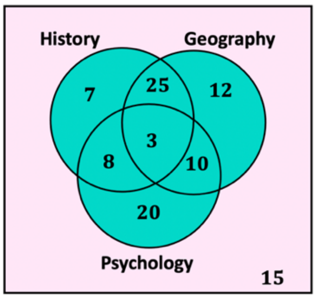


The total of these is equal to 30

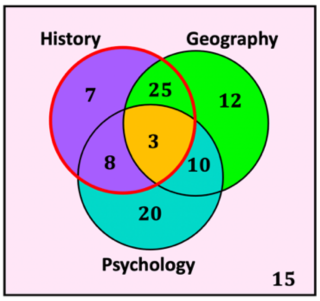
$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$
$$x + 26 = 30$$
$$x = 4$$

Hence the number of people who play all three sports is 4

21)



We choose all the options out of the red History circle for the first case, but we have to take them in cases since they affect Geography differently



$$\left(\frac{7+8}{100}\times\frac{25+12}{99}\right)+\left(\frac{25}{100}\times\frac{24+12}{99}\right)+\left(\frac{3}{100}\times\frac{25+12}{99}\right)=\frac{87}{550}$$