Probability 1: Venn Diagrams

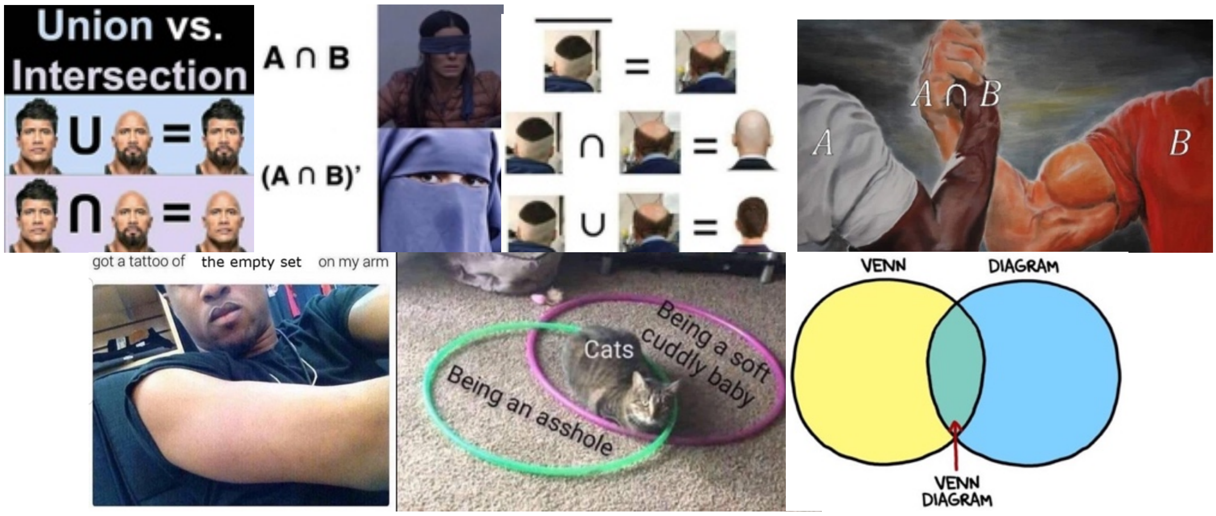


Table of Contents

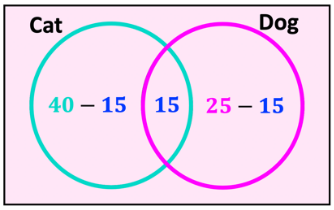
This is a long worksheet to cater for students that want extra practice. If you want a shortcut, but still be sure to cover one of each type then follow the pink highlighted questions.

1 Bronze

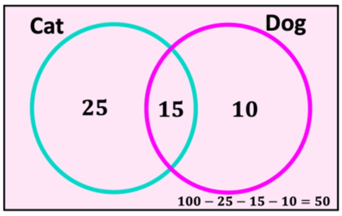
1.1 Without Set Notation

1)

40 own a cat.
25 own a dog
15 own a cat and a dog



Let's simplify the numbers



i. $\frac{25+15+10}{100} = \frac{50}{100} = \frac{1}{2}$

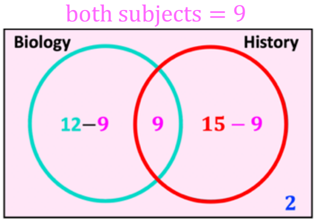
ii. $\frac{25+10}{100} = \frac{35}{100} = \frac{7}{20}$

iii. $\frac{15}{25+15} = \frac{3}{8}$

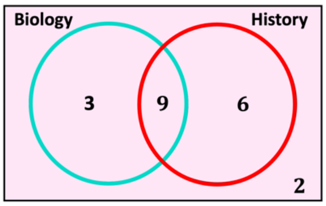
iv. $\frac{10}{15+10} = \frac{2}{5}$

2)

2 students study neither Biology nor History
20 students in total so $20 - 2 = 18$ study one of the subjects
12 study biology
15 student history
$12 + 15 - \text{both subjects} = 18$
$\text{both subjects} = 9$



Simplify

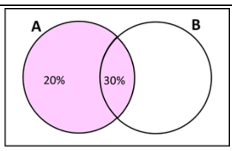


i. $\frac{9}{20}$

ii. $\frac{9}{3+9} = \frac{9}{12} = \frac{3}{4}$

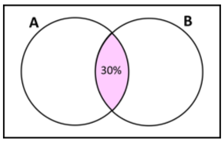
3)

i.



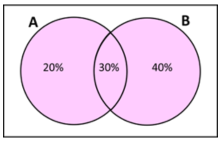
20% + 30% = 50% = 0.50

ii.



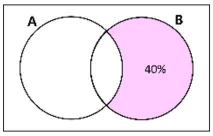
30% = 0.30

iii.



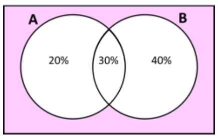
20% + 30% + 40% = 90% = 0.90

iv.



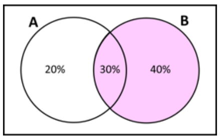
40% = 0.40

v.



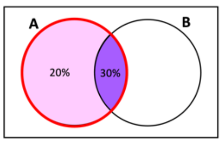
100% - 20% - 30% - 40% = 100% - 90% = 10% = 0.10

vi.



30% + 40% = 70% = 0.70

vii.



$$\frac{30\%}{20\% + 30\%} = \frac{30\%}{50\%} = \frac{3}{5} = 0.6$$

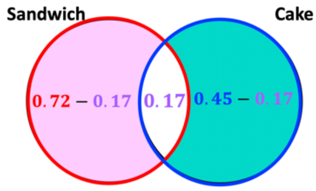
4)

We know all probabilities must add to one. The customer must either have a sandwich or a cake (i.e. there is no option to have neither so the outside is empty) so the inside of the Venn diagram must add to 1.

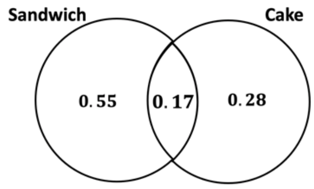
P(sandwich)= 0.72

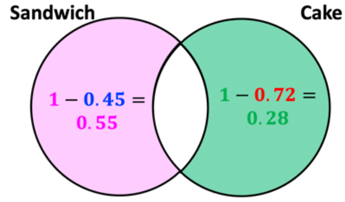
P (cake)= 0.45

Way 1:	**Way 2:**
1 = only a sandwich + only a cake + both 1 = (sandwich – both) + (cake – both) + both $1 = \text{sandwich} + \text{cake} - \text{both}$ $1 = 0.72 + 0.45 - \text{both}$ $\text{both} = 0.17$	$\text{Only a sandwich} = 1 - \text{cake}$ $\text{Only a sandwich} = 1 - 0.45 = 0.55$ $\text{Only a cake} = 1 - \text{sandwich}$ $\text{Only a cake} = 1 - 0.72 = 0.28$



Simplify



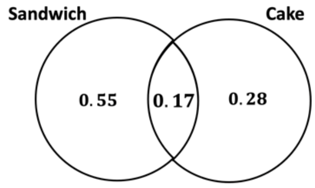


We can find the inside part now since probabilities add to 1

$0.55 + 0.28 + \text{both} = 1$

$\text{both} = 1 - 0.55 - 0.28 = 0.17$

So, we have



i. 0.17

ii. 0.55

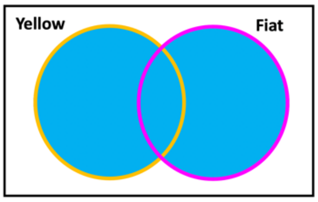
iii. probability each customer buys a cake $= 0.17 + 0.28 = 0.45$

200 expected customers a day

So expected number of cakes $= 0.45 \times 200 = 90$

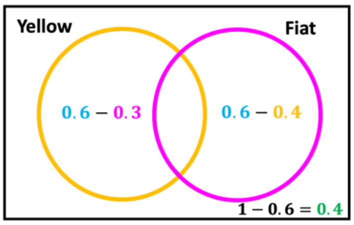
5)

P(yellow or a fiat) $= 0.6$

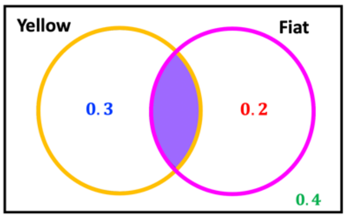


P(yellow)= 0.4

P(fiat)= 0.3



simplify



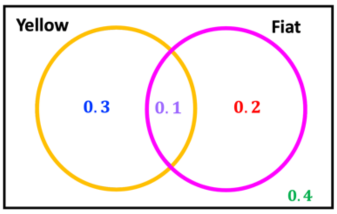
To find what the purple shaded region represents we use the fact that the sum of all probabilities = 1

0.3 + 0.2 + 0.4 + both = 1

Re-arranging to find out what both represents

$\text{both} = 1 - 0.3 - 0.2 - 0.4 = 0.1$

we can fill this into the Venn diagram



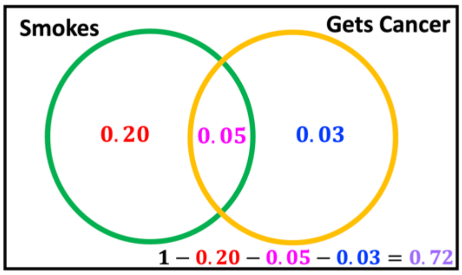
Probability of **not** yellow fiat = 1 – probability of yellow and a fiat
= 1 – 0.1
= 0.9

Note: Could also have done $0.3 + 0.2 + 0.4 = 0.9$

6)

Smokes and gets cancer = 0.05
Smoke and does not get cancer = 0.20
Does not smoke and gets cancer= 0.03

The sum of all probabilities $= 1 \Longrightarrow$ does not smoke and does not get cancer = 1− 0.20 – 0.05 – 0.03 = 0.72



i. $\frac{\text{gets cancer and is smoker}}{\text{total smokers}} = \frac{0.05}{0.2+0.05} = \frac{0.05}{0.25} = \frac{1}{5} = 0.2$

ii. $\frac{\text{does not get cancer and is a smoker}}{\text{total smokers}} = \frac{0.2}{0.2+0.05} = \frac{0.2}{0.25} = \frac{4}{5} = 0.8$

iii. $\frac{\text{non-smokers who get cancer}}{\text{total non-smokers}} = \frac{0.03}{0.03+0.72} = \frac{0.03}{0.75} = 0.04$

iv. $\frac{\text{does not get cancer and is non-smoker}}{\text{total non-smokers}} = \frac{0.72}{0.03+0.72} = \frac{0.72}{0.75} = 0.96$

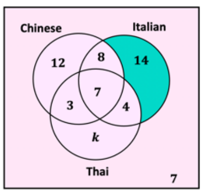
7)

Total must add to 60

So $60 = 12 + 8 + 14 + 3 + 7 + 4 + k$

So $k = 5$

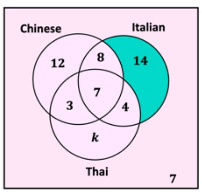
i.



Probability they like Italian

$$\frac{8+14+7+4}{60} = \frac{33}{60} = \frac{11}{20}$$

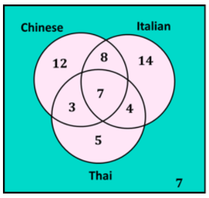
ii.



Probability they like only Italian

$$\frac{14}{60} = \frac{7}{30}$$

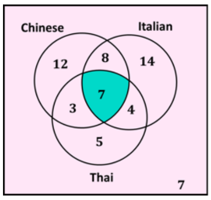
iii.



Probability they like none of these choices

$$\frac{7}{60}$$

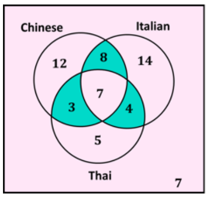
iv.



Probability they like all of these choices

$$\frac{7}{60}$$

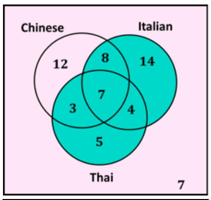
v.



Probability they like only two out of three

$$\frac{8+3+4}{60}=\frac{15}{60}=\frac{1}{4}$$

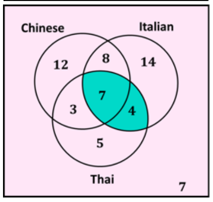
vi.



Probability they like Thai or Italian

$$\frac{8+14+7+3+4+5}{60}=\frac{41}{60}$$

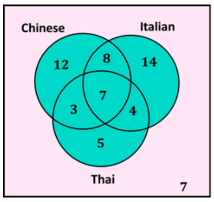
vii.



Probability they like Italian and Thai

$$\frac{7+4}{60}=\frac{11}{60}$$

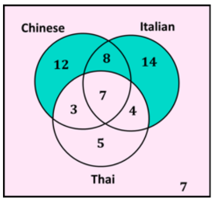
viii.



Probability they like at least one of these choices

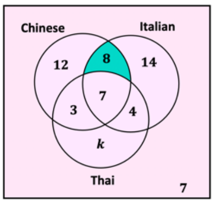
$$\frac{12+8+14+7+3+4+5}{60}=\frac{53}{60}$$

ix.



Probability they like Chinese or Italian, but not Thai

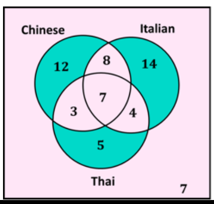
$$\frac{12+8+14+7+3+4+5}{60}=\frac{53}{60}$$



Probability they like Chinese and Italian, but not Thai

$$\frac{8}{60}=\frac{53}{60}$$

x.

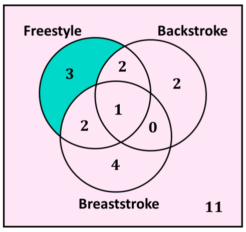


Probability they like exactly one of the three

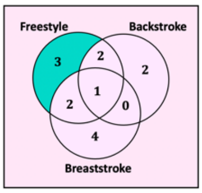
$$\frac{12+5+14}{60}=\frac{31}{60}$$

8)

Swimmers who swim none of these events = $25 - 3 - 2 - 2 - 2 - 1 - 0 - 4 = 25 - 14 = 11$

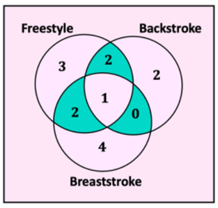


i.



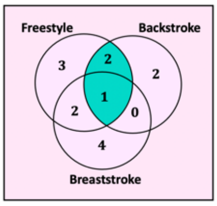
P(swims only freestyle) = $\frac{3}{25}$

ii.



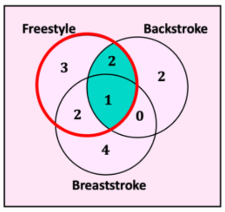
P(swims exactly 2 events) = $\frac{2+2+0}{25} = \frac{4}{25}$

iii.



P(swims freestyle and backstroke) = $\frac{2+1}{25} = \frac{3}{25}$

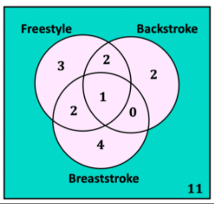
v.



P(swims backstroke given swims freestyle)

$= \frac{\text{swims backstroke and freestyle}}{\text{swims freestyle}} = \frac{2+1}{3+2+1+2} = \frac{3}{8}$

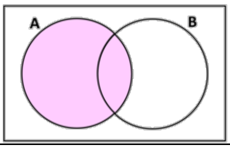
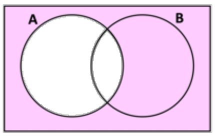
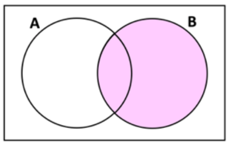
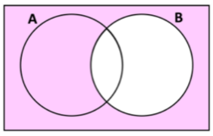
vi.



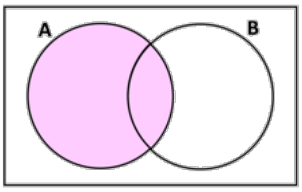
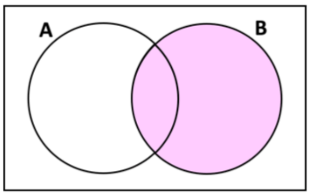
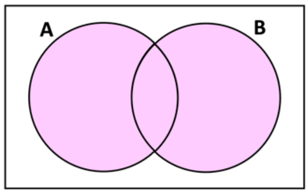
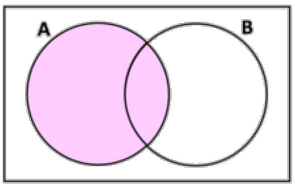
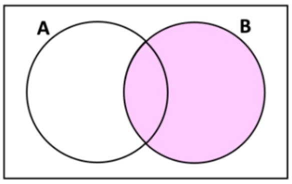
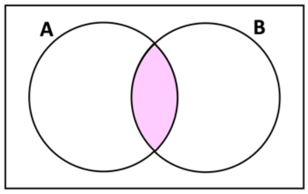
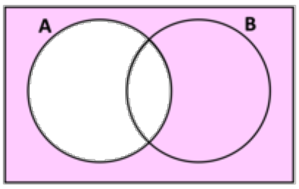
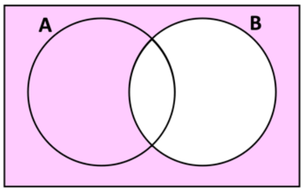
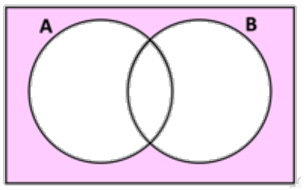
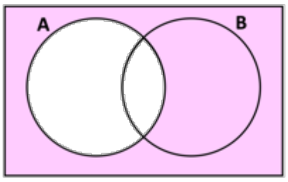
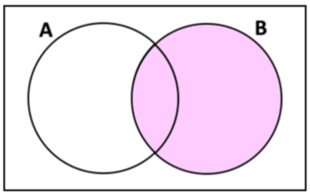
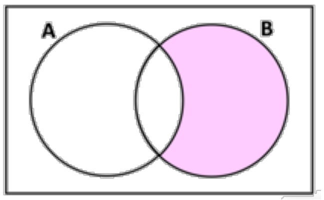
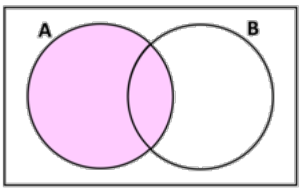
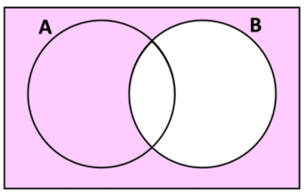
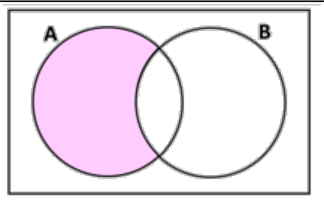
P(does not swim freestyle, backstroke or breaststroke) = $\frac{11}{25}$

1.2 With Set Notation

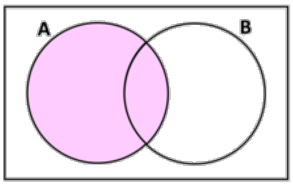
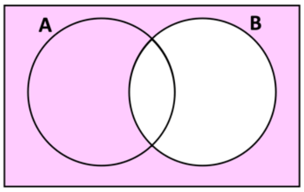
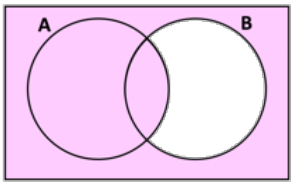
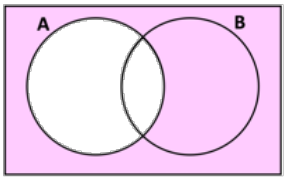
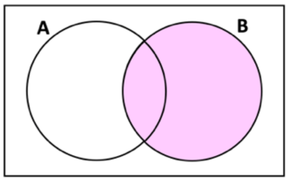
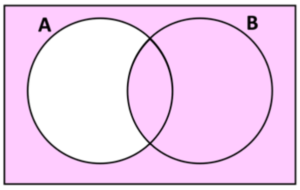
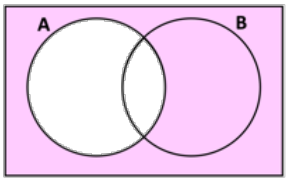
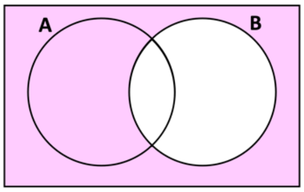
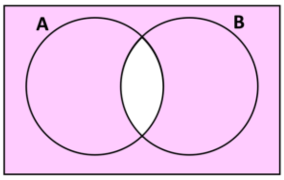
9)

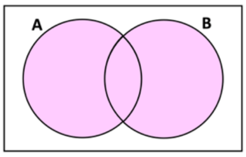
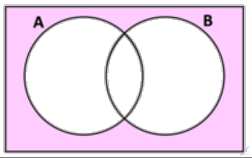
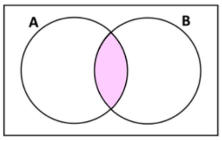
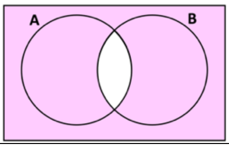
i. $P(A)$ 	iii. $P(A')$ 
ii. $P(B)$ 	iv. $P(B')$ 

10)

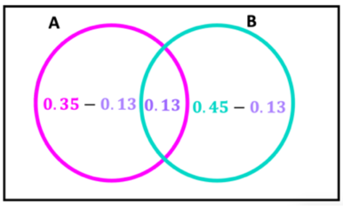
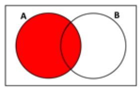
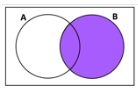
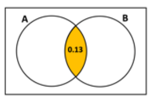
	Step 1	Step 2	Explained	Answer
$\boldsymbol{P(A \cup B)}$	 $P(A)$ =Everything in A	 $P(B)$ =Everything in B	This is a ∪ so we shade everything that is shaded in each diagram i.e. we **merge/combine** the shading in the diagrams together	
$\boldsymbol{P(A \cap B)}$	 P(A) = Everything in A	 P(B) = Everything in B	This is a ∩ so we shade everything that is shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams	
$\boldsymbol{P(A' \cap B')}$	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	This is a ∩ so we shade everything that is shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams	
$\boldsymbol{P(A' \cap B)}$	 $P(A')$ = Everything NOT in A	 $P(B)$ = Everything in B	This is a ∩ so we shade everything that is shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams	
$\boldsymbol{P(A \cap B')}$	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	This is a ∩ so we shade everything that is shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams	

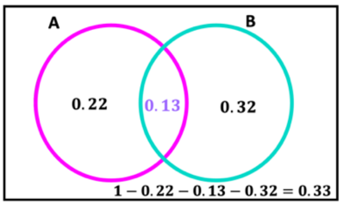
www.mymathscloud.com © MyMathsCloud

$\boldsymbol{P(A \cup B')}$	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	This is a ∪ so we shade everything that is shaded in each diagram i.e. we **merge/combine** the shading in the diagrams together	
$\boldsymbol{P(A' \cup B)}$	 $P(A')$ = Everything NOT in A	 $P(B)$ =Everything in B	This is a ∪ so we shade everything that is shaded in each diagram i.e. we **merge/combine** the shading in the diagrams together	
$\boldsymbol{P(A' \cup B')}$	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	This is a ∪ so we shade everything that is shaded in each diagram i.e. we **merge/combine** the shading in the diagrams together	

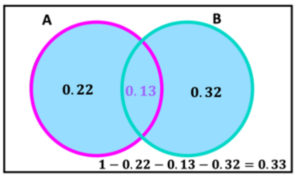
$P(A \cup B)'$:	$P(A \cap B)'$:
Shade $P(A \cup B)$ first  The ' tells us to find everything not in that region 	Shade $P(A \cap B)$ first  The ' tells us to find everything not in that region 

11)

Way 1: use a combination of a Venn diagram and formulae (if necessary)	**Way 2: Use the symmetries of a Venn diagram**
P(A)= 0.35 P(B)= 0.45 P(A∩B)= 0.13  simplify	We are told $P(A) = 0.35$. This means the shaded region to the right must represent 0.35  We are told $P(B) = 0.45$. This means the shaded region to the right must represent 0.45  We are told $P(A \cap B)$ =0.13. This means the shaded region to the right must represent 0.13 



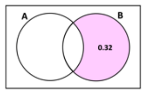
$P(A \cup B)$ is everything inside the circles



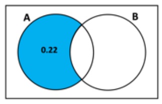
$P(A \cup B) = 0.22 + 0.13 + 0.32 = 0.67$
$P(A \cap B') = 0.22$

Note: We could have found $P(A \cup B)$ with just using the addition formula from the beginning without any Venn diagram as this question is easy:
Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Let's plug in everything we know
$P(A \cup B) = 0.35 + 0.45 - 0.13$
$P(A \cup B) = 0.67$

So, the remaining pink part must be
$0.45 - 0.13 = 0.32$

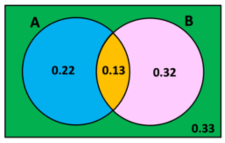


We also can find the remaining blue part
$0.35 - 0.13 = 0.22$



We can now we can fill in the all the pieces of the Venn diagram

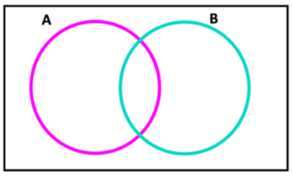
$1 - 0.32 - 0.13 - 0.22 = 0.33$



Reading off the diagram:
$P(A \cup B) = 0.22 + 0.13 + 0.32 = 0.67$
$P(A \cap B') = 0.22$

12)

P(A)=0.2, P(B)=0.5



We can't fill in anything into the Venn diagram yet

Let's use the addition formula
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

i.

If A and B are mutually exclusive: $P(A \cap B) = 0$
The addition formula above for $P(A \cup B)$ can be updated
$P(A \cup B) = 0.2 + 0.5 - 0 = 0.7$

ii.

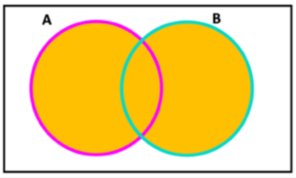
If A and B are independent: $P(A \cap B) = P(A) \times P(B)$
$P(A \cap B) = 0.2 \times 0.5 = 0.1$

The addition formula above for $P(A \cup B)$ can be updated
$P(A \cup B) = 0.2 + 0.5 - 0.1 = 0.6$

13)

i.

$P(A) = 0.38$
$P(B) = 0.42$
$P(A \cup B) = 0.56$



We can't fill into the Venn diagram yet

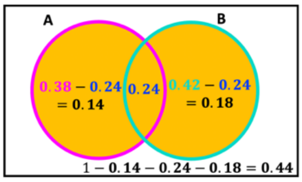
Let's use the addition formula first to find the intersection $P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.56 = 0.38 + 0.42 - P(A \cap B)$$

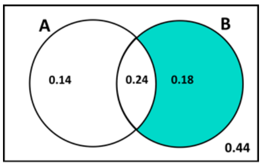
Re-arrange for $P(A \cap B)$

$$P(A \cap B) = 0.38 + 0.42 - 0.56 = 0.24$$

Now we can fill into the Venn diagram

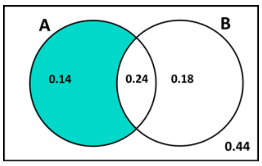


ii.



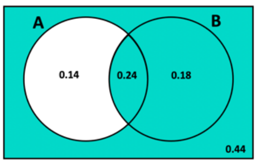
$P(A' \cap B) = 0.18$

iii.



$P(A \cap B') = 0.14$

iv.



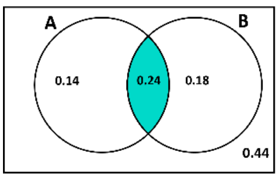
$P(A' \cup B) = 0.18 + 0.24 + 0.44 = 0.8$

Note: Could also have used the addition formula instead and adapted it by replacing A with A'

$$P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$$
$$= (0.18 + 0.44) + (0.24 + 0.18) - 0.18 = 0.86$$

v.

$P(A \cap B)$:



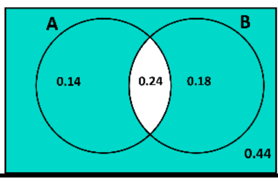
Remember the ' means everything NOT in $A \cap B$

$P(A \cap B)' = 0.14 + 0.18 + 0.44 = 0.76$

Note: Could also have used

$P(A \cap B)' = 1 - P(A \cap B) = 1 - 0.24 = 0.76$

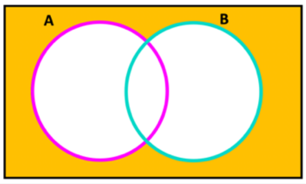
$P(A \cap B)'$:



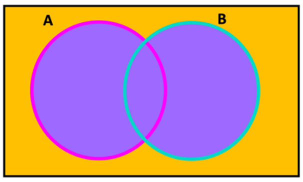
14)

i.

$$P(A) = \frac{11}{36}$$
$$P(B) = \frac{1}{6}$$
$$P(A \cup B)' = \frac{21}{36}$$



We can't fill into the Venn diagram yet, but we can find out what $P(A \cup B)$ is below by using $P(A \cup B)'$ and the fact that probabilities add to 1



$$P(A \cup B)' = 1 - P(A \cup B)$$
$$P(A \cup B) = 1 - \frac{21}{36} = \frac{15}{36}$$

Let's use the addition formula first to find the intersection

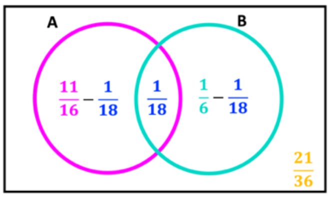
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{15}{36} = \frac{11}{36} + \frac{1}{6} - P(A \cap B)$$

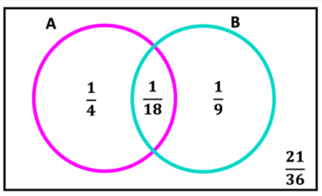
$$P(A \cap B) = \frac{11}{36} + \frac{1}{6} - \frac{15}{36} = \frac{2}{36} = \frac{1}{18}$$

So, we know $P(A) = \frac{11}{36}$, $P(B) = \frac{1}{6}$, $P(A \cup B) = \frac{15}{36}$, $P(A \cap B) = \frac{1}{18}$

we can fill in the Venn diagram now



Simplify



ii. $P(A' \cap B) = \frac{1}{9}$

iii. $P(A \cap B') = \frac{1}{4}$

iv. $P(A' \cup B) = \frac{1}{18} + \frac{1}{9} + \frac{21}{36} = \frac{3}{4}$

Note: Could also have used the addition formula instead and adapted it by replacing A with A'

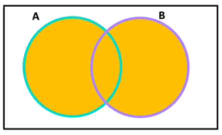
$P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = \left(\frac{1}{9} + \frac{21}{36}\right) + \left(\frac{1}{18} + \frac{1}{9}\right) - \frac{1}{9} = \frac{3}{4}$

1.2.1 Independence/ Mutually Exclusive

15)

Way 1: Use a combination of a Venn diagram and formulae

$P(A) = 0.4$, $P(B) = 0.55$ and $P(A \cup B) = 0.7$



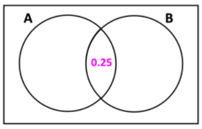
We can't fill into the Venn diagram yet

Let's use the addition formula first to find the intersection: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.40 + 0.55 - P(A \cap B)$

$P(A \cap B) = 0.25$

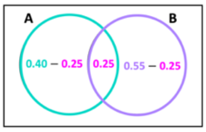
let's update the Venn diagram with $P(A \cap B) = 0.25$



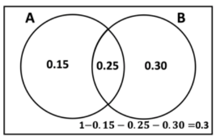
Now we can fill in the rest of the Venn diagram

P(A)=0.40

P(B)=0.55



Let's simplify the numbers

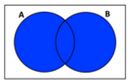


Reading off the diagram:

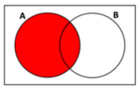
$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

Way 2: Use the symmetries of Venn diagram

we are told P(A∪B)=0.7. This means the shaded region to the right must represent 0.7

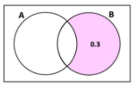


we are also told $P(A) = 0.4$. This means the shaded region to the right must represent 0.4

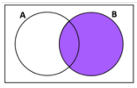


so, the remaining part must be

$0.7 - 0.4 = 0.3$

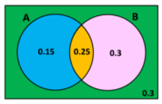


we are also told $P(B) = 0.55$. This means the shaded region to the right must represent 0.55



So, we can now we can fill in the all the pieces of the Venn diagram

$0.55 - 0.3 = 0.25$



$0.4 - 0.25 = 0.15$

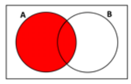
$1 - 0.3 - 0.25 - 0.15 = 0.3$

Reading off the diagram:

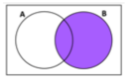
$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

Way 3: Use the symmetries of Venn diagram (same method as way 2, just a different order)

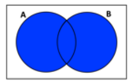
we are also told $P(A) = 0.4$. This means the shaded region to the right must represent 0.4



we are also told $P(B) = 0.55$. This means the shaded region to the right must represent 0.55

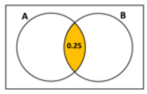


we are told P(A∪B)=0.7. This means the shaded region to the right must represent 0.7



The red and purple added together double counts the middle region. We can find this region by subtracting the red and purple regions from the blue region

$0.7 - 0.40 - 0.55 = 0.25$

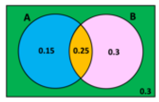


So, we can now we can fill in the all the pieces of the Venn diagram

$0.55 - 0.25 = 0.25$

$0.4 - 0.25 = 0.15$

$1 - 0.3 - 0.25 - 0.15 = 0.3$



Reading off the diagram:

$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

A and B are independent if $P(A \cap B) = P(A) \times P(B)$

Let's find each individual part and see whether the above result is true

$$P(A \cap B) = 0.25$$
$$P(A) = 0.40$$
$$P(B) = 0.55$$
$$P(A) \times P(B) = 0.40(0.55) = 0.22$$

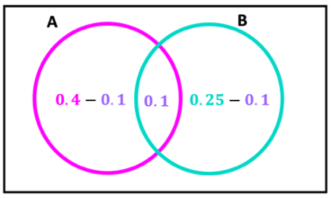
$P(A \cap B) \neq P(A) \times P(B)$ since $0.25 \neq 0.22$

Therefore, the events are not independent

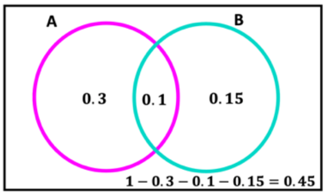
16)

A and B are independent so we can use a formula for this: $P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.25 = 0.1$

So, we have
P(A)=0.4
P(B)=0.25
$P(A \cap B) = 0.1$



simplify



Way 1: Read off Venn Diagram

i. $P(A \cup B)$ is the inside of the circles
$P(A \cup B) = 0.3 + 0.1 + 0.1 = 0.55$
ii. $P(A \cap B)$ is the middle part common to both circles
$P(A \cap B)$ =0.1
iii. $P(A \cap B')$ means in A, but not in B, so this is the half moon/crescent on the left
$P(A \cap B') = 0.3$
iv. $P(A' \cap B')$ is the outside region
$P(A' \cap B') = 0.45$
v.$P(A \cup B') = 0.3 + 0.1 + 0.45 = 0.85$
Note: Could also have used the addition formula instead and adapted it by replacing B with B'
$P(A \cup B') = P(A) + P(B') - P(A \cap B') \quad = (0.3 + 0.1) + (0.3 + 0.45) - 0.3 = 0.85$

Way 2: Use Formulae
i. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.25 - 0.1 = 0.55$
ii. $P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.25 = 0.1$
Note: we could only multiply since the events are independent
iii. $P(A \cap B') = P(A) \times \left(1 - P(B)\right) = 0.4 \times (1 - 0.25) = 0.4 \times 0.75 = 0.3$
Note: we could only multiply since the events are independent
iv. $P(A' \cap B') = \left(1 - P(A)\right) \times (1 - P(B)) = (1 - 0.4) \times (1 - 0.25) = 0.6 \times 0.75 = 0.45$
Note: we could only multiply since the events are independent
v. $P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.4 + 0.75 - 0.3 = 0.85$

17)

i. A and B are independent so $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.5 = 0.15$
ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.5 - (0.3 \times 0.5) = 0.3 + 0.5 - 0.15 = 0.65$
iii. $P(A \cap B) \neq 0$ so events A and B are **not** mutually exclusive

1.2.2 Conditional

18)

We don't need to draw a Venn diagram here as we have everything we need given in the question

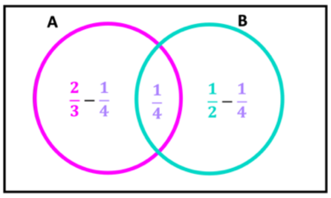
Use the formula $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$P(B|A) = \frac{0.12}{0.2} = \frac{3}{5} = 0.6$$

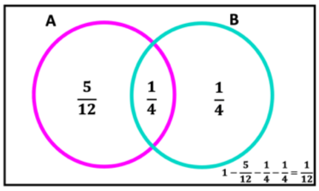
19)

Way 1: Use a combination of Venn diagram and formulae

$\text{P(A)} = \frac{2}{3}$
$\text{P(B)} = \frac{1}{2}$
$\text{P(A}\cap\text{B)} = \frac{1}{4}$



simplify



$P(A \cup B)$ is the inside of the circles

$$P(A \cup B) = \frac{5}{12} + \frac{1}{4} + \frac{1}{4} = \frac{11}{12}$$

$P(A \cap B)'$ is the outside region

$$P(A \cap B)' = \frac{1}{12}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

Way 2: Use formulae only

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{11}{12}$$

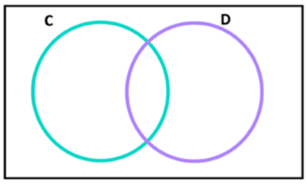
$$P(A \cap B)' = 1 - P(A \cap B) = 1 - \frac{11}{12} = \frac{1}{12}$$

$P(A|B)$ has a formula

$$\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

20)

$P(C) = 06$
$P(D) = 0.3$

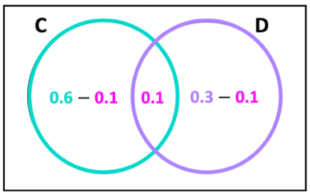


We can't fill into the Venn diagram yet as we don't have enough information.

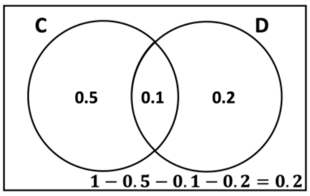
Let's use the addition formula

$$P(C \cup D) = P(C) + P(D) - P(C \cap D)$$
$$0.8 = 0.6 + 0.3 - P(C \cap D)$$
$$P(C \cap D) = 0.1$$

So, the Venn Diagram looks like



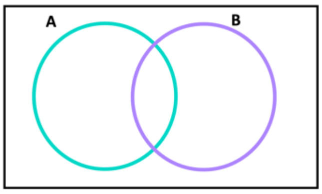
Simplify



So $P(D|C') = \frac{P(D \cap C')}{P(C')} = \frac{0.2}{0.4} = \frac{1}{2} = 0.5$

21)

$$P(A) = 0.5$$
$$P(B) = 0.4$$



We can't fill into the Venn diagram yet as we don't have enough information.

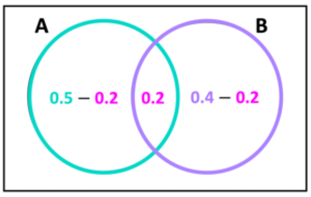
We are given A and B independent:

What does this mean in terms of the formula? It means $P(A \cap B) = P(A) \times P(B)$

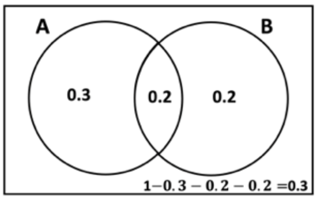
$$P(A \cap B) = 0.5 \times 0.4 = 0.2$$

Now we have enough info to fill into the Venn diagram

$$P(A) = 0.5$$
$$P(B) = 0.4$$
$$P(A \cap B) = 0.2$$



Let's simplify the numbers



We want $P(B|A)$

This has a formula. $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.2}{0.3+0.2} = \frac{0.2}{0.5} = \frac{2}{5}$

2 Silver

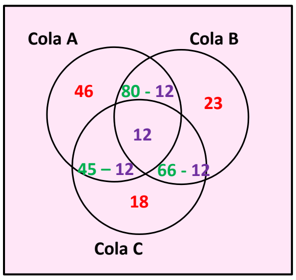
2.1 Without Set Notation

22)

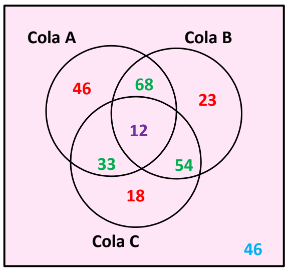
46 like only Cola A, 23 like only Cola B, 18 like only Cola C

80 like both Colas A and B, 66 like both Colas A and C, 45 like both Colas B and C

12 like all three Colas



Simplify



Note: 300 in total, so $300 - 46 - 68 - 23 - 12 - 33 - 54 - 18 = 46$ don't like any Cola.

i. Likes all three colas $= \frac{12}{300} = \frac{1}{25}$

ii. Likes colas A and B $= \frac{68+12}{300} = \frac{80}{300} = \frac{4}{15}$

iii. Likes Cola A or Cola B$= \frac{46+68+23+12+33+54}{300} = \frac{236}{300} = \frac{59}{75}$

iv. Does not like Cola A or B or C$= \frac{46}{300} = \frac{23}{150}$

v. Likes at least 2 Colas $= \frac{68+12+33+54}{300} = \frac{167}{300}$

vi. Likes only 1 Cola $= \frac{46+23+18}{300} = \frac{87}{300} = \frac{29}{100}$

vii. Likes B and does not like C $= \frac{68+23}{300} = \frac{91}{300}$

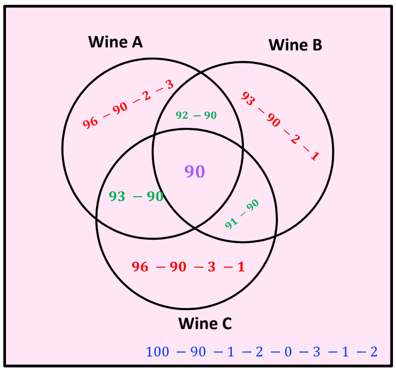
23)

90 like all 3

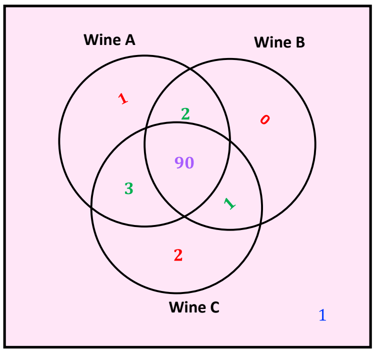
92 like A and B, 91 like B and C, 93 like A and C

96 like A, 93 like B, 96 like C

100 in total



Simplify



i. $\frac{1}{100}$

ii. $\frac{1+3}{100}=\frac{4}{100}$

iii. $\frac{1+2+0}{100}=\frac{3}{100}$

iv. $\frac{1+2+0}{100}=\frac{3}{100}$

v. $\frac{2+3+1}{100}=\frac{6}{100}$

vi. $\frac{1+2+0+90+3+1+2}{100}=\frac{99}{100}$

vii. $\frac{1}{100}$

viii. $\frac{2}{100}$

ix. $\frac{1+2+90+3+1}{100}=\frac{97}{100}$

x. $\frac{1+2+0}{100}=\frac{3}{100}$

xi. $\frac{\text{like A and C}}{\text{total that like A}}=\frac{3+90}{90+1+2+3}=\frac{93}{96}$

xii. $\frac{\text{likes A or B or both AND doesn't like A}}{\text{likes A or B or both}} = \frac{\text{likes B and doesn't like A}}{\text{likes A or B or both}} = \frac{0+1}{1+2+0+90+3+1} = \frac{1}{97}$

xiii. A and B are independent if the probability that a person likes A multiplied by the probability a person likes B equals the probability that a person likes A and B.

probability a person likes A $= \frac{90+1+2+3}{100} = \frac{96}{100}$

probability a person likes B $= \frac{90+0+2+1}{100} = \frac{93}{100}$

probability a person likes A and B $= \frac{90+2}{100} = \frac{92}{100}$

$\frac{96}{100} \times \frac{93}{100} \neq \frac{92}{100}$

So, A and B are **not** independent.

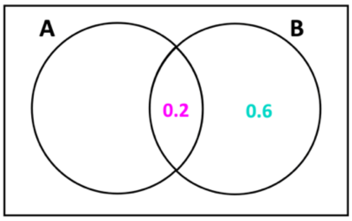
2.2 With Set Notation

2.3 Independence

24)

Way 1: Use a combination of Venn diagram and formulae

$P(A\cap B)=0.2$
$P(A'\cap B)=0.6$

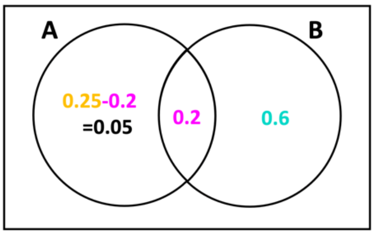


i.

$$P(B) = 0.2 + 0.6 = 0.8$$

ii.
Given A and B Independent:
$P(A \cap B) = P(A) \times P(B)$
$0.2 = P(A) \times (0.6 + 0.2)$
$P(A) = \frac{0.2}{0.8} = 0.25$

Let's update the Venn diagram



$P(A \cup B) = 0.05 + 0.2 + 0.6 = 0.85$

Way 2: use formulae only
i. $P(B) = P(A \cap B) + P(A' \cap B) = 0.2 + 0.6 = 0.8$

ii. A and B are independent so $P(A \cap B) = P(A) \times P(B)$

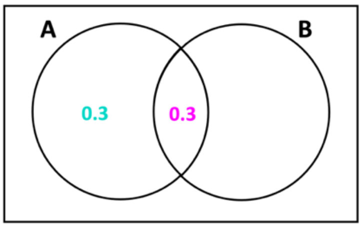
$$\text{so } 0.2 = P(A) \times 0.8$$
$$P(A) = \frac{0.2}{0.8} = 0.25$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.8 - 0.2 = 0.85$$

25)

Way 1: Use a combination of Venn diagram and formulae

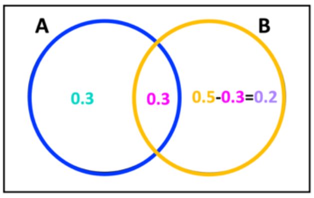
P(A∩ B)=0.3
P(A∩ B')=0.3



We can see that $P(A) = 0.3 + 0.3 = 0.6$
$P(A) = 0.6$

Given A and B Independent: We can user the independence formula
$P(A \cap B) = P(A) \times P(B)$
$P(A \cap B) = 0.6\ P(B)$
$0.3 = 0.6 P(B)$
$P(B) = \frac{0.3}{0.6} = 0.5$

Let's update the Venn diagram



P(A∪ $B) = 0.3 + 0.3 + 0.2 = 0.8$

Way 2: use formulae only
A and B are independent so $P(A \cap B) = P(A) \times P(B)$ and $P(A \cap B') = P(A) \times P(B') = P(A) \times (1 - P(B))$
$$0.3 = P(A) \times P(B)$$
$$0.3 = P(A) \times (1 - P(B))$$

Now we have a pair of simultaneous equations
We can let $P(A) = x, P(B) = y$ so it looks more familiar.

$$(1)\ 0.3 = xy$$
$$(2)\ 0.3 = x(1 - y)$$

$$\frac{(1)}{(2)} \quad 1 = \frac{y}{1 - y}$$

$$y - 1 = y$$

$$2y = 1$$

$$y = \frac{1}{2} = 0.5 = P(B)$$

Substitute this back into equation (1)

$$(1)\ x = P(A) = \frac{0.3}{y} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6$$

So $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.5 - 0.3 = 0.8$

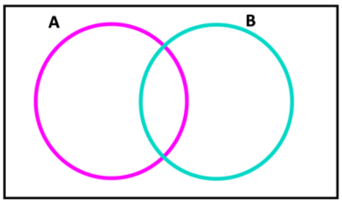
2.3.1.1 Conditional

26)

Way 1: Use a combination of Venn diagram and formulae

$P(A) = \frac{2}{5}$

$P(B) = \frac{11}{20}$



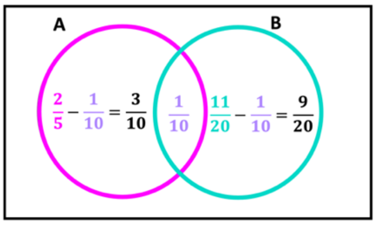
We don't have enough to fill into the Venn diagram yet

Given $P(A|B) = \frac{2}{11}$

We can use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{P(A \cap B)}{\frac{11}{20}} = \frac{2}{11}$$

$$P(A \cap B) = \frac{2}{11} \times \frac{11}{20} = \frac{1}{10}$$



$$P(A \cup B) = \frac{3}{10} + \frac{1}{10} + \frac{9}{20} = \frac{17}{20}$$

Way 2: Use formulae only

Given $P(A|B) = \frac{2}{11}$

Let's use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{2}{11} = \frac{P(A \cap B)}{\frac{11}{20}}$$

$$P(A \cap B) = \frac{2}{11} \times \frac{11}{20} = \frac{1}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{5} + \frac{11}{20} - \frac{1}{10} = \frac{17}{20}$$

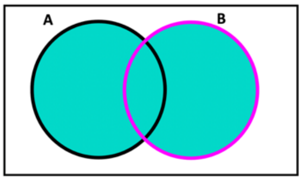
$P(A|B) = \frac{2}{11}$ but $P(A) = \frac{2}{5}\frac{2}{11} \neq \frac{2}{5}$ so A and B are **not** independent.

27)

Way 1: Use a combination of Venn diagram and formulae

P(B)= $\frac{1}{2}$

P(A∪ B)= $\frac{13}{20}$



We don't have enough to fill into the Venn diagram yet

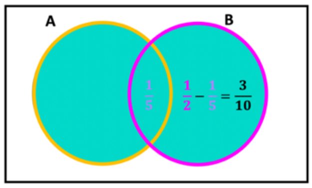
Given P(A|B)= $\frac{2}{5}$

We can use the conditional formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\frac{P(A \cap B)}{\frac{1}{2}} = \frac{2}{5}$$

$$P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

We can now fill in the middle part



P(A)= $\frac{13}{20} - \frac{3}{10} = \frac{7}{20}$

Let's use the conditional formula to find $P(B|A)$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

P(A'∩ B) $= \frac{3}{10}$

Way 2: Use formulae only

Given P(A|B)= $\frac{2}{5}$

Let's use the conditional formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\frac{2}{5} = \frac{P(A \cap B)}{\frac{1}{2}}$$

$$P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{13}{20} = P(A) + \frac{1}{2} - \frac{1}{5}$$

$$P(A) = \frac{13}{20} + \frac{1}{5} - \frac{1}{2} = \frac{7}{20}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

$$P(B) = P(A \cap B) + P(A' \cap B)$$

$$\frac{1}{2} = \frac{1}{5} + P(A' \cap B)$$

$$P(A' \cap B) = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

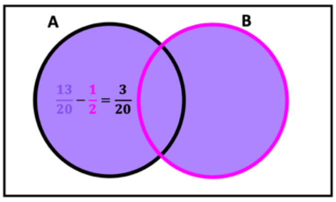
28)

Way 1: Use a combination of Venn diagram and formulae

$P(B)=\frac{1}{2}$

$P(A|B)=\frac{2}{5}$

$P(A \cup B)=\frac{13}{20}$



We don't have enough to fill in the whole Venn diagram yet

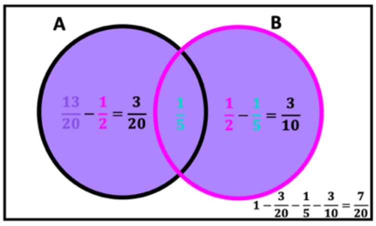
Let's use the conditional formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{5} = \frac{P(A \cap B)}{\frac{1}{2}}$$

$$P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

Now we can fill in the rest of the Venn diagram



$$P(A) = \frac{3}{20} + \frac{1}{5} = \frac{7}{20}$$

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{3}{20}}{\frac{3}{20} + \frac{7}{20}} = \frac{3}{10}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

$$P(A' \cup B) = \frac{3}{10} + \frac{7}{20} + \frac{1}{5} = \frac{17}{20}$$

Note: Could also have used the addition formula instead and adapted it by replacing A with A'

$$P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = \left(\frac{3}{10} + \frac{7}{20}\right) + \left(\frac{1}{5} + \frac{3}{10}\right) - \frac{3}{10} = \frac{17}{20}$$

Way 2: Use formulae only

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$\frac{2}{5} = \frac{P(A \cap B)}{\frac{1}{2}}$$
$$P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

$$P(B) = P(A \cap B) + P(A' \cap B)$$
$$\frac{1}{2} = \frac{1}{5} + P(A' \cap B)$$
$$P(A' \cap B) = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{13}{20} = P(A) + \frac{1}{2} - \frac{1}{5}$$
$$P(A) = \frac{13}{20} - \frac{1}{2} + \frac{1}{5} = \frac{7}{20}$$

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{\frac{7}{20} - \frac{1}{5}}{1 - \frac{1}{2}} = \frac{\frac{3}{20}}{\frac{1}{2}} = \frac{3}{10}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

$$P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = \left(1 - P(A)\right) + P(B) - P(A' \cap B)$$
$$= \left(1 - \frac{7}{20}\right) + \frac{1}{2} - \frac{3}{10} = \frac{13}{20} + \frac{1}{2} - \frac{3}{10} = \frac{17}{20}$$

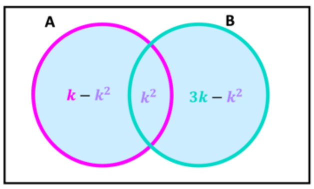
2.3.1.2 With Algebra

29)

Way 1: Use a combination of Venn diagram and formulae

i.

P(A)= k
P(B)= $3k$
P(A∩B)=k^2
P(A∪B)=0.5



We can build an equation based on knowing the blue shaded region is 0.5

$$k - k^2 + k^2 + 3k - k^2 = \frac{1}{2}$$
$$k^2 - 4k + \frac{1}{2} = 0$$
$$2k^2 - 8k + 1 = 0$$

Using calculator/ quadratic formula, $k = 3.871$ or $k = 0.129$

Since k represents a probability, $k \leq 1$ so $k = 0.129$ is the only solution

ii.

$$P(A' \cap B) = 3k - k^2 = 3(0.129) - 0.129^2 = 0.370$$

Way 2: Use formulae only

i.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{1}{2} = k + 3k - k^2$$

$$2k^2 - 8k + 1 = 0$$

Using calculator/ quadratic formula, $k = 3.871$ or $k = 0.129$
Since k represents a probability, $k \leq 1$ so $k = 0.129$ is the only solution

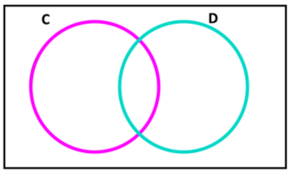
ii.

$$P(B) = P(A \cap B) + P(A' \cap B)$$
$$P(A' \cap B) = P(B) - P(A \cap B) = 3k - k^2 = 3 \times 0.129 - 0.129^2 = 0.370$$

30)

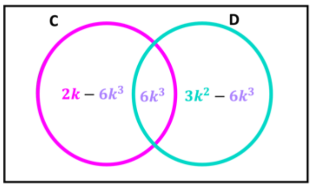
Way 1: Use a combination of Venn diagram and formulae

P(C)=$2k$
P(D)=$3k^2$



Not enough info to fill in the Venn diagram yet

We can use the fact that C and D are independent: $P(C \cap D) = P(C) \times P(D)$
$P(C \cap D) = 2k \times 3k^2 = 6k^3$



Now we can fill into the Venn diagram

i. $6k^3$
ii. $6k^3 = 0.162$
$k^3 = 0.027$
$k = \sqrt[3]{0.027} = 0.3$
iii. $P(C'|D) = \frac{P(C' \cap D)}{P(D)} = \frac{3k^2 - 6k^3}{3k^2} = \frac{1-2k}{1} = \frac{1-0.6}{1} = 0.4$

Way 2: Use formulae only

i.C and D are independent so $P(C \cap D) = P(C) \times P(D)$
$$P(C \cap D) = 2k \times 3k^2 = 6k^3$$

ii. $0.162 = 6k^3$

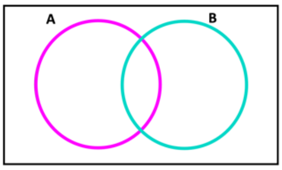
$$k^3 = \frac{0.162}{6} \; so \; k = \sqrt[3]{\frac{0.162}{6}} = 0.3$$

iii. $P(C'|D) = \frac{P(C' \cap D)}{P(D)} = \frac{P(D) - P(C \cap D)}{P(D)} = \frac{3k^2 - 6k^3}{3k^2} = \frac{1-2k}{1} = \frac{1-0.6}{1} = 0.4$

31)

Way 1: Use a combination of Venn diagram and formulae

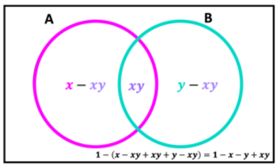
P(A)$= x$
P(B)$= y$



We don't have enough to fill into the Venn diagram yet

Given that events A and B are independent so let's use the formula P$(A \cap B) = P(A) \times P(B)$
P$(A \cap B) = P(A) \times P(B) = xy$

Now we can fill into the Venn diagram



i.$P(A \cap B) = P(A) \times P(B) = xy$
ii.$P(A \cup B) = x - xy + xy + y - xy = x + y - xy$
iii.$P(A \cup B') = x - xy + xy + 1 - x - y + xy = 1 - y + xy$
Note: Could also have used the addition formula instead and adapted it by replacing B with B′
$P(A \cup B') = P(A) + P(B') - P(A \cap B')$
$= (x - xy + xy) + (x - xy + 1 - x - y + xy) - (x - xy)$
$= x + 1 - y - x + xy$
$= 1 - y + xy$

Way 2: Use formulae only

i. $P(A \cap B) = P(A) \times P(B) = xy$
ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = x + y - xy$
iii. $P(A \cup B') = P(A) + P(B') - P(A \cap B')$
$= P(A) + \left(1 - P(B)\right) - \left(P(A) \times \left(1 - P(B)\right)\right)$
$= x + (1 - y) - x(1 - y) = x + 1 - y - x + xy = 1 - y + xy$

3 Gold

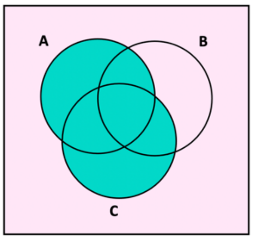
3.1 With Set Notation

32)

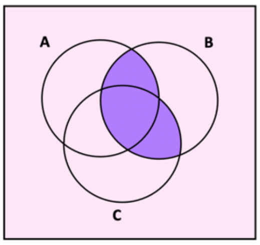
i.

$P((A \cup C) \cap B)$

We deal with what is in the bracket first. This means we shade $P(A \cup C)$ first



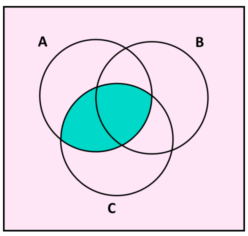
Now we deal with the $\cap B$ part. This means we choose the parts that are also in B i.e. remove anything that is not in B



ii.

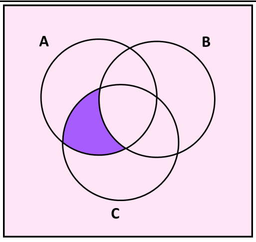
$P(B' \cap (A \cap C))$

We deal with what is in the bracket first. So we shade $P(A \cap C)$ first



Now we deal with the $B' \cap$ part

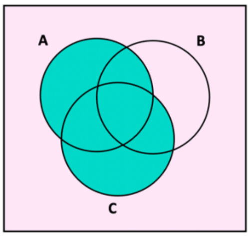
This means we are choosing the parts of $A \cap C$ that are also NOT in B, ie. Remove the bits that are in B



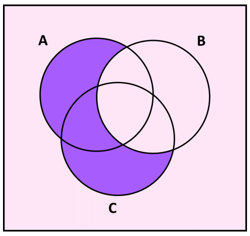
iii.

$P(B' \cap (A \cup C))$

deal with the bracket first so shade $A \cup C$



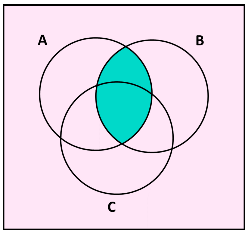
Now consider $B' \cap$ so chose the parts that are also NOT in B; ie. Remove everything that is in B



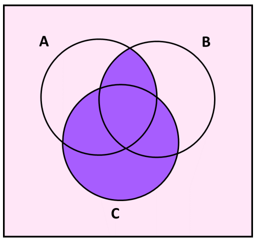
iv.

$P((A \cap B) \cup C)$

consider the bracket first; shade $A \cap B$



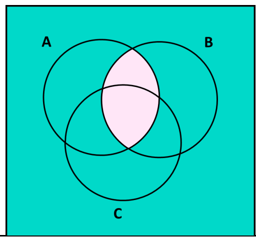
Now deal with $\cup\, C$ so also shade everything in C



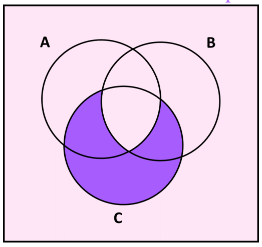
v.

$P(A' \cup B') \cap C)$

Dealing with the bracket first, we want everything that is not in A and everything that is not in B

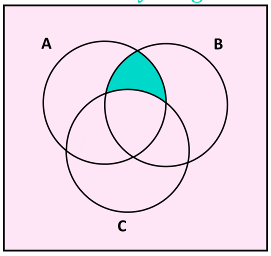


Now think about $\cap C$ which means we want the parts of this that are also in C

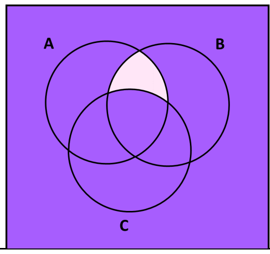


vi. $P((A \cap B \cap C')')$

start with the bracket, we want everything that's in A and B but not in C



Now deal with the $'$, so we want to shade the opposite of what we just found



33)

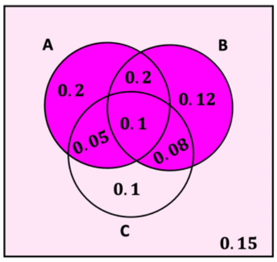
i. $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.2+0.1}{0.2+0.1+0.12+0.08} = \frac{0.3}{0.5} = 0.6$

ii. $P(C|A') = \frac{P(C\cap A')}{P(A')} = \frac{0.1+0.08}{0.12+0.08+0.1+0.15} = \frac{0.18}{0.45} = 0.4$

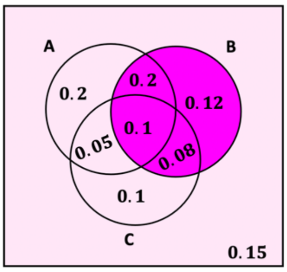
iii. $P(B|A \cup B) = \frac{P(B\cap(A\cup B))}{P(A\cup B)}$

Let's look at the numerator $P(B \cap (A \cup B))$

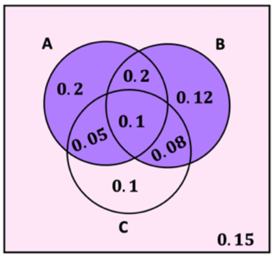
Shade $P(A \cup B)$ first



Now choose the parts that are also in B



Let's look at the denominator $P(A \cup B)$

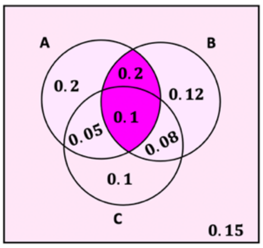


$$P(B|A \cup B) = \frac{P(B \cap (A \cup B))}{P(A \cup B)} = \frac{0.2 + 0.1 + 0.12 + 0.08}{0.2 + 0.05 + 0.2 + 0.1 + 0.12 + 0.08} = \frac{0.5}{0.75} = \frac{2}{3}$$

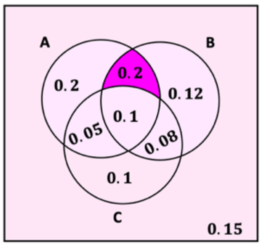
ii. $P\big((A \cap B)\big|C'\big) = \frac{P((A \cap B) \cap C')}{P(C')}$

Let's look at the numerator $P((A \cap B) \cap C')$

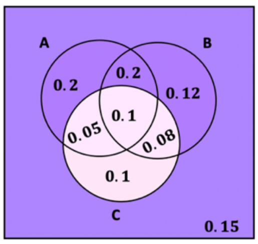
Shade $P(A \cap B)$ first



Now choose the parts that are also NOT in C



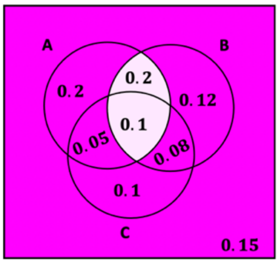
Let's look at the denominator P(C')



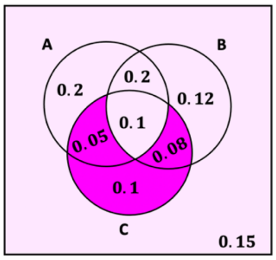
$$P\big((A \cap B)\big|C'\big) = \frac{P(A \cap B \cap C')}{P(C')} = \frac{0.2}{0.2+0.2+0.12+0.15} = \frac{0.2}{0.67} = 0.299$$

iii. $P(C|A' \cup B') = P(C|A' \cup B') = \frac{P\big(C \cap (A' \cup B')\big)}{P(A' \cup B')}$

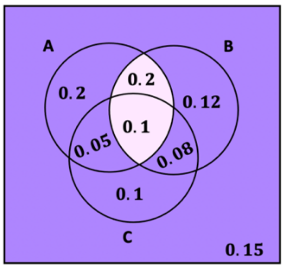
Let's look at the numerator $P\big(C \cap (A' \cup B')\big)$
Shade $P(A' \cup B')$ first



Now choose the parts that are also in C



Let's look at the denominator $P(A' \cup B')$



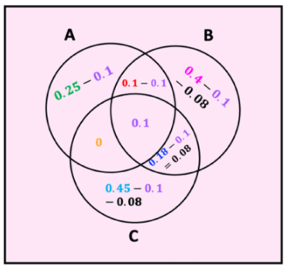
$$\frac{P\big(C\cap(A'\cup B')\big)}{P(A'\cup B')} = \frac{0.05+0.08+0.1}{0.2+0.05+0.08+0.12+0.1+0.15} = \frac{0.23}{0.7} = 0.329$$

3.1.1 Independence/Mutually Exclusive

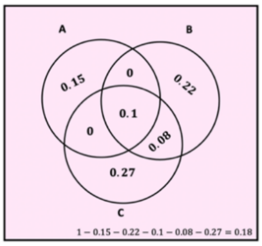
34)

i.

$P(A) = 0.25$
$P(B) = 0.4$
$P(C) = 0.45$
$P(A \cap B \cap C) = 0.1$
$A \cap B' \cap C = \emptyset$
$P(A \cap B) = P(A)P(B) = 0.25(0.4) = 0.1$
$P(B \cap C) = P(B)P(C) = 0.4(0.45) = 0.18$

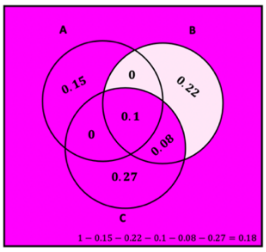


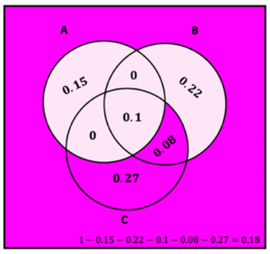
We can simplify the numbers



ii.

$P(A' \cap (B' \cup C))$
Shade $P(B' \cup C)$ first – left diagram
Now choose the parts that are also NOT in A – right diagram

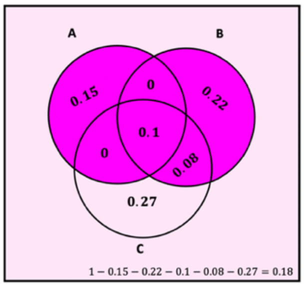


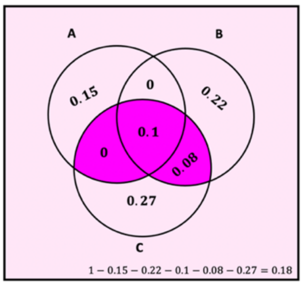


$$0.08 + 0.27 + 0.18 = 0.53$$

iii.

$P(A \cup B) \cap C)$
Shade $P(A \cup B)$ first – left diagram
Now choose the bits that are also in C – right diagram

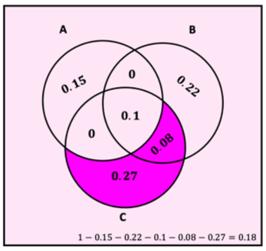




$$0 + 0.1 + 0.08 = 0.18$$

iv.

If independent $P(A' \cap C) = P(A')P(C)$
$P(A') = 1 - 0.25 = 0.75$ (since A was given in question)
$P(C) = 0.45$ (since C was given in question)



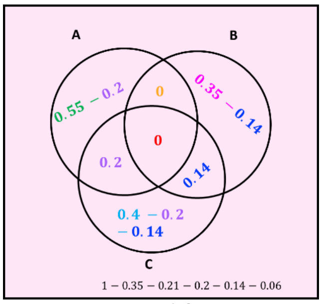
$P(A' \cap C) = 0.27 + 0.08 = 0.35$
$P(A')P(C) = 0.75(0.45) = 0.3375$
$0.35 \neq 0.3375$
So not independent

35)

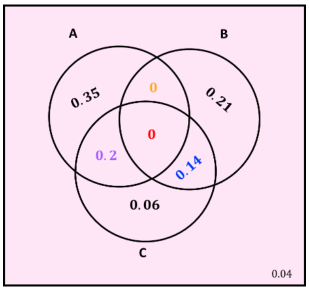
$$P(A) = 0.55$$
$$P(B) = 0.35$$
$$P(C) = 0.4$$
$$P(A \cap C) = 0.2$$
$$P(A \cap B) = 0$$
$$P(B \cap C) = P(B)P(C) = 0.35(0.4) = 0.14$$

A and B are mutually exclusive so $P(A \cap B) = 0$

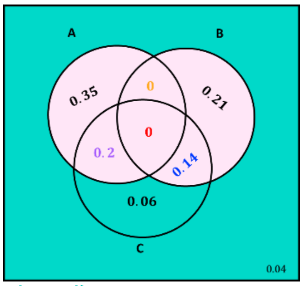
i.



Simplify

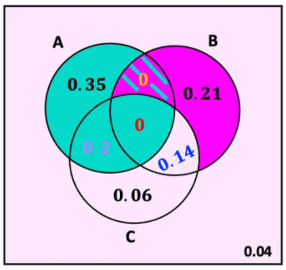


ii.



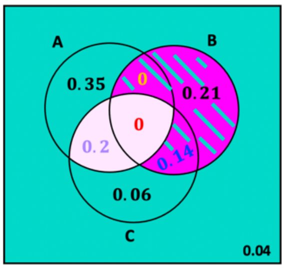
$P(A' \cap B') = 0.06 + 0.04 = 0.1$

iii.



$P(A \cup (B \cap C')) = 0.35 + 0.2 + 0.21 = 0.76$

iv.

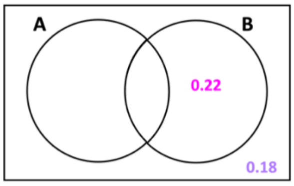


iv. $P((A \cap C)' \cup B) = 0.35 + 0.21 + 0.14 + 0.06 + 0.04 = 0.8$

3.1.2 With Algebra

36)

$P(A' \cap B) = 0.22$
$P(A' \cap B') = 0.18$



There isn't much that can be filled into the Venn diagram

Let's use: P(AIB)=0.6

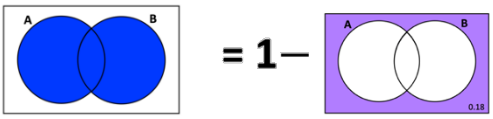
This means $\frac{P(A \cap B)}{P(B)} = 0.6$

This doesn't help either

This is hard since can't fill into the formula in order fill out the rest of hte Venn diagram. So, we must use the symmetries below instead.

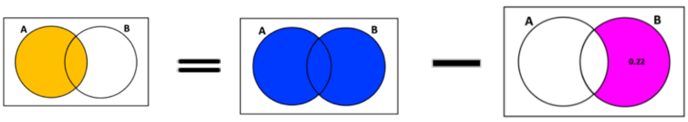
i. and ii.

Way 1:



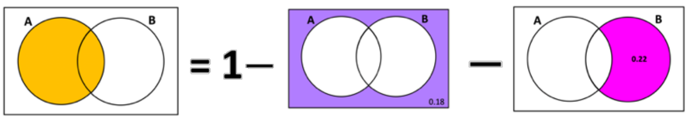
So $P(A \cup B) = 1 - 0.18 = 0.82$

Now we can find $P(A)$ since

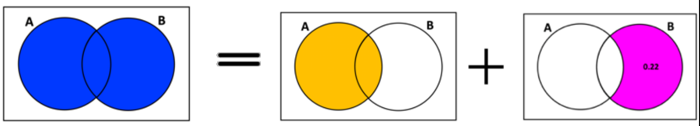


Find $P(A) = 0.82 - 0.22 = 0.6$

Way 2:



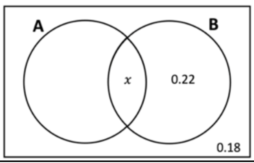
$P(A) = 1 - 0.18 - 0.22 = 0.6$



$P(A \cup B) = 0.6 + 0.22 = 0.82$

iii.

We still don't have enough info to fill all numbers into the Venn diagram. To get around this, lets we call the intersection part x



P(AIB)=0.6

$$\frac{x}{x+0.22} = 0.6$$
$$x = 0.33$$

Note: Or you can use that $P(A'|B) = \frac{P(A' \cap B)}{P(B)}$

$$1 - 0.6 = \frac{0.22}{P(B)}$$
$$P(B) = 0.55$$
$$P(A \cap B) = 0.55 - 0.33 = 0.22$$

iv.

$P(A) = 0.6, P(B) = 0.22 + 0.33 = 0.55$

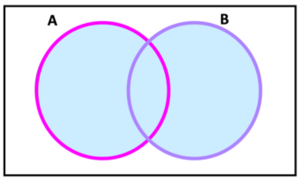
$P(A \cap B) = 0.33$

$0.33 = 0.6(0.55)$ therefore independent

37)

P(A) $= \frac{1}{4}$

$P(A \cup B) = \frac{2}{3}$



Not enough into to fill in the Venn diagram

A and B are independent: we can use the formula $P(A \cap B) = P(A)P(B)$

This doesn't help since we don't have enough info to fill in

We can also use the addition formula:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ can be updated as

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$ since independent

$\frac{2}{3} = \frac{1}{4} + P(B) - \frac{1}{4}P(B)$

Solve for $P(B)$. Call if x if it helps look a bit more familiar to the types of equations you're used to solving.

$$\frac{2}{3} - \frac{1}{4} = x - \frac{1}{4}x$$

$$\frac{5}{12} = \frac{3}{4}x$$

$$x = \frac{\frac{5}{12}}{\frac{3}{4}} = \frac{5}{9}$$

So $P(B) = x = \frac{5}{9}$

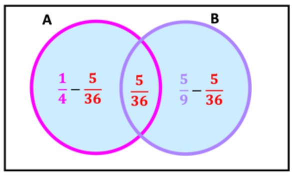
Now we have enough info to fill into the Venn diagram

P(A)$= \frac{1}{4}$

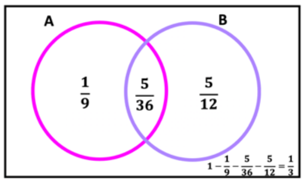
$P(A \cup B) = \frac{2}{3}$

$P(B) = \frac{5}{9}$

A and B are independent so $P(A \cap B) = P(A) \times P(B) = \frac{1}{4} \times \frac{5}{9} = \frac{5}{36}$



Simplify

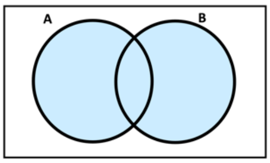


From Venn Diagram, $P(A' \cap B) = \frac{5}{12}$

$$P(B'|A) = \frac{P(B' \cap A)}{P(A)} = \frac{\frac{1}{9}}{\frac{1}{9} + \frac{5}{36}} = \frac{\frac{1}{9}}{\frac{1}{4}} = \frac{4}{9}$$

38)

P(A∪B)=0.52



We don't have enough info to fill into the Venn diagram

Let's use some formulae

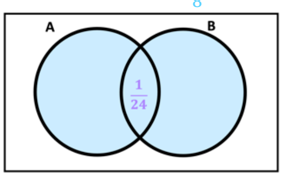
Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ can be updated as
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$ since A and B are independent
$0.52 = \mathrm{P(A)} + 2P(A) - \mathrm{P(A)}[2P(A)]$

Solve for $P(A)$. Call if x if you want
$0.52 = x + 2x - x(2x)$
$0.52 = x + 2x - 2x^2$
$2x^2 - 3x + 0.52 = 0$
$x = \frac{13}{10}, \frac{1}{5}$

So $P(A) = x = \frac{1}{5} = 0.2$, since probabilities cannot be greater than 1
$P(B) = 2P(A) = 2\left(\frac{1}{5}\right) = \frac{2}{5} = 0.4$

39)

P(A∩B) $= \frac{1}{24}$
P(A∪B) $= \frac{3}{8}$



We don't have enough info to fill into the Venn diagram

Let's use some formulae

Independent: P(A∩B)=P(A)P(B)

$\frac{1}{24}$ =P(A)P(B)

Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ can be updated as

$\frac{3}{8} = P(A) + P(B) - \frac{1}{24}$

Solve simultaneously

We can say $P(A) = x$ and $P(B) =$ y if you prefer

$\frac{1}{24} = xy$

$x = \frac{1}{24y}$

$\frac{3}{8} = x + y - \frac{1}{24}$

$x + y = \frac{5}{12}$

$\frac{1}{24y} + y = \frac{5}{12}$

$1 + 24y^2 = 10y$

$24y^2 - 10y + 1 = 0$

$y = \frac{1}{4}, \frac{1}{6} \Longrightarrow x = \frac{1}{6}, \frac{1}{4}$

So $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{6}$ or vice versa

40)

i.

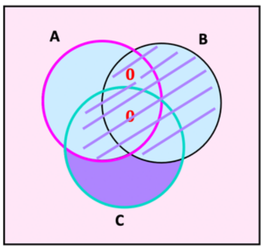
A and B are mutually exclusive: $P(A \cap B) = 0$

P(A)=0.2

P(C)=0.3

P(A∪ $B) = 0.4$

P(B∪ C)=0.34



As you can see the Venn diagram doesn't help

Let's use some formulae

A and B are mutually exclusive so $P(A \cap B) = 0$

We can update the addition formula with this:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.4 = 0.2 + P(B) - 0$$
$$P(B) = 0.4 - 0.2 = 0.2$$

We can use the addition formula again:

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$
$$0.34 = 0.2 + 0.3 - P(B \cap C)$$
$$P(B \cap C) = 0.2 + 0.3 - 0.34 = 0.16$$

ii. B and C are independent if $P(B|C) = P(B)$ and $P(C|B) = P(C)$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.16}{0.2} = 0.8$$

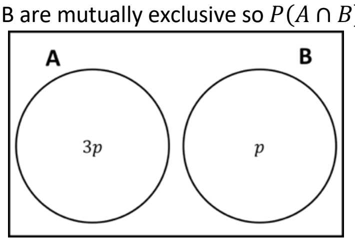
So $P(B|C) \neq P(B)$ since $0.2 \neq 0.8$

41)

i.

$P(B) = p \; so \; P(A) = 3p$

A and B are mutually exclusive so $P(A \cap B) = 0$



ii.

Sum of probabilities must be between 0 and 1

$$0 \leq 3p + p \leq 1$$
$$0 \leq 4p \leq 1$$
$$0 \leq p \leq \frac{1}{4}$$
$$0 \leq P(B) \leq \frac{1}{4}$$

iii.

If C and D are independent, $P(C \cap D) = P(C)P(D)$

$$P(C|D) = 3P(C)$$
$$P(C|D) = \frac{P(C \cap D)}{P(D)} = 3P(C)$$

So $P(C \cap D) = 3P(C)P(D) \neq P(C)P(D)$ since $P(C) \neq 0$

So C and D are **not** independent.

iv.

$P(C \cap D) = \frac{1}{2}P(C)$ and $P(C' \cap D') = \frac{7}{10}$

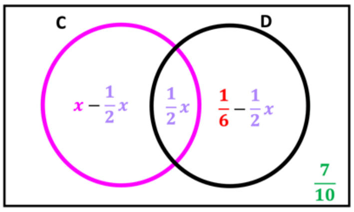
Let $P(C) = x$

So $P(C \cap D) = \frac{1}{2}x$

$$P(C' \cap D') = \frac{7}{10}$$

From part(ii) $P(D) = \frac{P(C \cap D)}{3P(C)} = \frac{\frac{1}{2}x}{3x} = \frac{1}{6}$

Venn diagram:



Probabilities must sum to 1

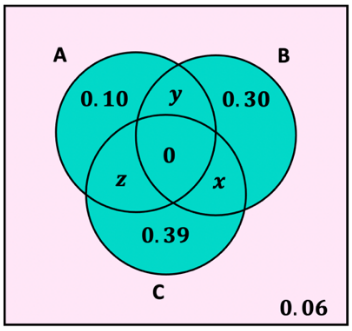
$$x-\frac{1}{2}x+\frac{1}{2}x+\frac{1}{6}-\frac{1}{2}x+\frac{7}{10}=1$$
$$\frac{1}{2}x+\frac{13}{15}=1$$
$$\frac{1}{2}x=\frac{2}{15}$$
$$x=P(C)=\frac{4}{15}$$

4 Diamond

4.1 With Set Notation

4.2 With Algebra

42)



We can form 3 equations and have 3 unknowns so we can find each unknown

A and B mutually exclusive: $x + 0 = 0$

A and C independent: $z = (0.10 + y + z)(z + x + 0.39)$

Probabilities add to make 1: $0.1 + z + y + 0.3 + x + 0.39 + 0.06 = 1$

$$z + y + 0.85 = 1$$
$$z + y = 0.15$$

$z = (0.10 + y + z)(z + x + 0.39)$ becomes

$$z = (0.10 + 0.15)(z + 0 + 0.39)$$
$$z = 0.25(z + 0.39)$$
$$z = 0.25z + 0.0975$$
$$0.75z = 0.0975$$
$$z = 0.13$$

$$0.13 + y = 0.15$$
$$y = 0.02$$

$$x = 0, y = 0.02, z = 0.13$$

43)

i.

Pairs of mutually exclusive events are where there is no overlap on the Venn diagram:

A and C

B and D

C and D

ii.

$$\mathbf{P(B) = 0.4}$$
$$p + 0.07 + 0.24 = 0.4$$
$$p = 0.09$$

iii.

$$\boldsymbol{P(A \cap B) = P(A) \times P(B)}$$
$$(0.24 + 0,.16 + q)0.4 = 0.24$$
$$0.24 + 0.16 + q = 0.6$$
$$q = 0.2$$

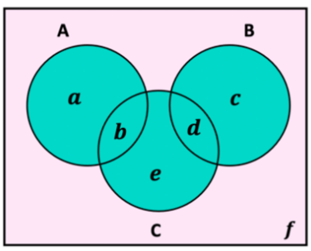
iv.

$$\boldsymbol{P\,(\,B'\,|\,C\,)\ =\ 0.64:}$$
$$\frac{P(B' \cap C)}{P(C)} = 0.64$$
$$\frac{r}{p + r} = 0.64$$
$$\frac{r}{0.09 + r} = 0.64$$
$$r = 0.64(0.09 + r)$$
$$r = 0.0576 + 0.64r$$
$$r - 0.64r = 0.0576$$
$$0.36r = 0.0576$$
$$r = 0.16$$

v. **Probabilities add to 1**

$$0.16 + 0.24 + 0.2 + 0.07 + 0.09 + 0.16 + s = 1$$
$$s = 1 - 0.92 = 0.08$$

44)



i. We can form equations based on all the given info

We can form 5 equations, but have 6 unknowns so we could never find each unknown, but that's ok

P(A)=0.2 : $a+b=0.2$

P(C)=0.3 : $b+d+e=0.3$

P(A∪ B) $=\mathbf{0.4}$: $a+b+c+d=0.4$
$a+b+c+d=0.4$
$0.2+c+d=0.4$
$c+d=0.2$

P(B∪ C)=0.34 : $b+c+d+e=0.34$
$b+c+d+e=0.34$
$0.3+c=0.34$
$c=0.04$

So $c+d=0.2$ becomes $0.04+d=0.2 \Longrightarrow d=0.16$

Probabilities add the 1: $a+b+c+d+e+f=1$ (we don't need this)

$P(B)=c+d=0.2$

$P(B\cap C)=d=0.16$

ii. Are B and C independent?
$P(B)=0.2$
$P(C)=0.3$
$P(B\cap C)=0.16$
$(0.2)(0.3)=0.06$
$0.06\neq 0.16$
Therefore $P(A\cap B)\neq P(A)P(B)$
not independent

45)

i. We can form equations based on all the given info
We can form 5 equations and 5 unknowns so we can find each unknown

P(A)=0.5 : $r+s=0.5$

P(B)=0.6 : $s+t+p=0.6$

P(C) $=\mathbf{0.25}$: $p+q=0.25$

Events B and C are independent : $(s+t+p)(p+q)=p$
$(0.6)(0.25)=p$
$p=0.15$

So $p+q=0.25$ becomes
$0.15+q=0.25$
$q=0.1$

ii. **Probabilities add the 1:** $r+s+t+p+0.1+0.08=1$
$r+s+t+p+0.1+0.08=1$

$$r + 0.6 + 0.1 + 0.08 = 1$$
$$r = 1 - 0.6 - 0.1 - 0.08$$
$$r = 0.22$$

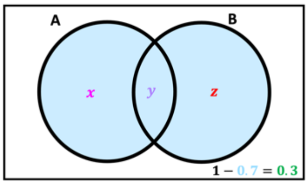
iii. $r + s = 0.5$ becomes
$$0.22 + s = 0.5$$
$$s = 0.5 - 0.22 = 0.28$$

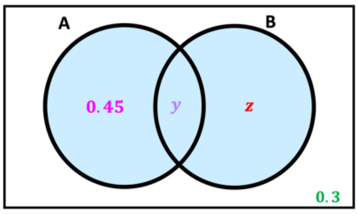
$s + t + p = 0.6$ becomes
$$0.28 + t + 0.15 = 0.6$$
$$t = 0.6 - 0.28 - 0.15$$
$$t = 0.17$$

iv. $P(A) = 0.5$ (given in question)
$P(B) = 0.6$ (given in question)
$$P(A \cap B) = s = 0.28$$
$$(0.5)(0.6) = 0.30$$
$$0.28 \neq 0.30$$
Therefore $P(A \cap B) \neq P(A)P(B)$
Not independent

$$\mathrm{P(B \mid A \cup C)} = \frac{P(B \cap A \cup C)}{P(A \cup C)} = \frac{s+p}{r+s+p+q} = \frac{0.28+0.15}{0.5+0.25} = 0.573$$

46)

Way 1: Use algebra	**Way 2: Without algebra (harder as have to think a bit more)**
We can't fill in anything to the Venn diagram Let's use the conditional formula; $P(A\|B') = 0.6$ $\frac{P(A \cap B')}{P(B')} = 0.6$ $P(A \cup B) = 0.7$ Call $P(A \cap B') = x$, $P(A \cap B) = y$, $P(A' \cap B) = z$ in the Venn diagram  $\frac{P(A \cap B')}{P(B')} = \frac{x}{x+0.3} = 0.6$ $x = 0.6x + 0.18$ $0.4x = 0.18$ $x = \frac{0.18}{0.4} = 0.45$ Fill this result back into the Venn diagram	$P(A'\|B') = \frac{P(A' \cap B')}{P(B')}$ $1 - P(A\|B') = \frac{P(A' \cap B')}{P(B')}$ $1 - 0.6 = \frac{0.3}{P(B')}$ $P(B') = 0.75$ $P(B) = 0.25$ **OR** $P(A\|B') = \frac{P(A \cap B')}{P(B')}$ $0.6 = \frac{P(A \cup B) - P(B)}{P(B')}$ $0.6 = \frac{0.7 - P(B)}{1 - P(B)}$ Solve for $P(B)$ $P(B) = 0.25$



Form equation based on knowing that $P(A \cup B) = 0.7$

$0.45 + y + z = 0.7$

$y + z = 0.25$

$P(B) = y + z = 0.25$

47)

	A	B	Total
Professional	740	380	1120
Skilled	275	90	365
Elementary	260	80	340
Total	1275	550	1825

i. $P(\text{employee is skilled}) = \frac{365}{1825} = \frac{1}{5}$

ii. $P(\text{lives in area B and is not a professional}) = \frac{90+80}{1825} = \frac{34}{365}$

iii. 65% of professional employees in both area A and B work from home

$$0.65 \times 1120 = 728$$

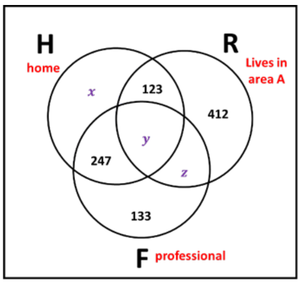
40% of skilled employees in both area A and B work from home

$$0.40 \times 365 = 146$$

5% of elementary employees in both area A and B work from home

$$0.05 \times 340 = 17$$

so total working from home $= 728 + 146 + 17 = 891$



Build equations to find $x, y \text{ and } z$

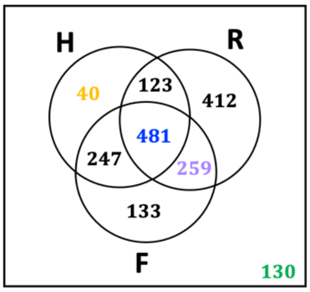
- 740 in A are professional: $y + z = 740$ (1)
- 891 work from home: $x + 123 + 247 + y = 891$

  $x + y = 521$ (2)
- 728 of professional work from home: $247 + y = 728$ so $y = 481$ (2)

Sub y into (2): $x + 481 = 521$ so $x = 40$

Sub y into (1): $481 + z = 790$ so $z = 259$

Total = 1825 so $1825 - 40 - 123 - 412 - 481 - 247 - 259 - 133 = 130$



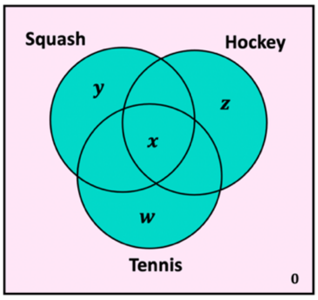
iv. Find $P(R' \cap F) = \frac{247+133}{1825} = \frac{380}{1825} = \frac{76}{365}$

v.Find $P((H \cup R)') = \frac{133+130}{1825} = \frac{263}{1825}$

vi. Find $P(F|H) = \frac{F \cap H}{H} = \frac{247+481}{40+123+481+247} = \frac{728}{891}$

48)

Way 1



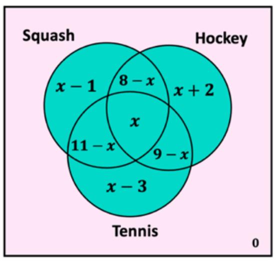
Let's find w, x, y and z in terms of x

We know the following

$$y + 11 + (8 - x) = 18$$
$$y = x - 1$$

$$w + 11 + (9 - x) = 17$$
$$w = x - 3$$

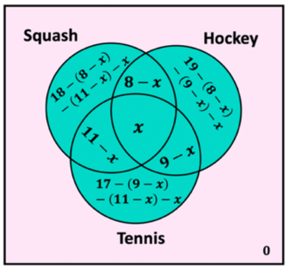
$$z + 8 + (9 - x) = 19$$
$$z = x + 2$$



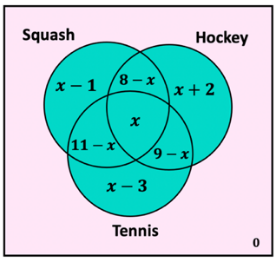
The total of these is equal to 30

$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$

Way 2



Let's simplify each part by collecting like terms



The total of these is equal to 30

$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$
$$x + 26 = 30$$
$$x = 4$$

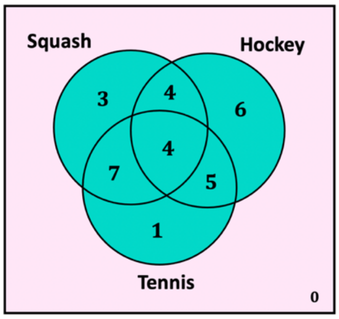
Hence the number of people who play all three sports is 4

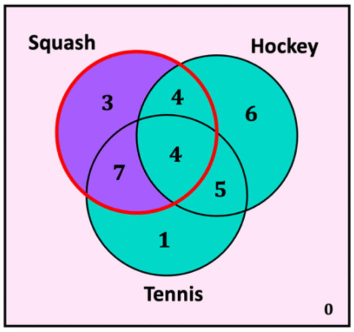
$$x + 26 = 30$$
$$x = 4$$

Hence the number of people who play all three sports is 4

ii.

Now that we know x we can fill out the rest of the diagram



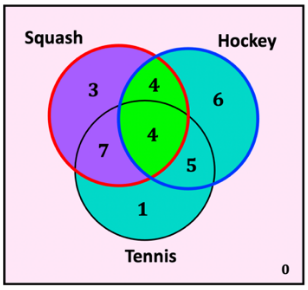


$$P(H'|S) = \frac{P(H' \cap S)}{P(S)} = \frac{7+3}{7+3+4+4} = \frac{10}{18} = \frac{5}{9}$$

i. Think back to when you first learnt probability and you were taking beads out of a bag without replacement. What happens in the first selection affected the numbers for what happens in the next selection. We care whether the first student played hockey or not as this will affect the numbers when selecting for the hockey. So, it should make sense that there are 2 cases you need to consider here. Either the first person plays hocket as well as squash, or the first person doesn't don't play hockey.

$$P(H|S) \times P(H) + P(H'|S) \times P(H)$$

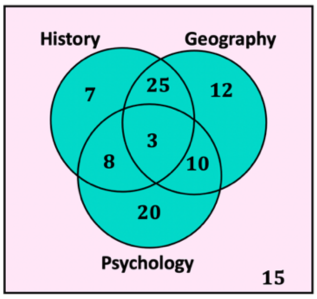
Where the first even in each is the first student and the second event in each is the second student



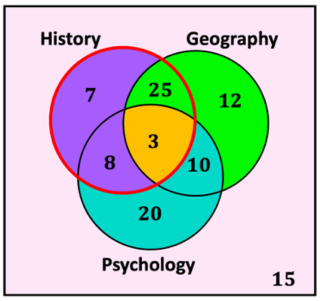
choose all the options out of the red Spanish circle for the first case, but we take them in cases (green and purple) since they affect Hockey differently

$$\left(\frac{4+4}{18}\right)\left(\frac{18}{29}\right) + \left(\frac{7+3}{18}\right)\left(\frac{19}{29}\right) = \frac{167}{261}$$

49)



We choose all the options out of the red History circle for the first case, but we have to take them in cases since they affect Geography differently



$$\left(\frac{7+8}{100}\times\frac{25+12}{99}\right)+\left(\frac{25}{100}\times\frac{24+12}{99}\right)+\left(\frac{3}{100}\times\frac{25+12}{99}\right)=\frac{87}{550}$$

50)

USE: $P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$ and events are independent so $P(X\cap Y)=P(X)\times P(Y)$

i. $P(A\cap B\cap C)=P\big((A\cap B)\cap C\big)=P(A\cap B)\times P(C)=\big(P(A)\times P(B)\big)\times P(C)=(xy)z=xyz$

ii. $P(A\cup B\cup C)=P\big((A\cup B)\cup C\big)$
$=P(A\cup B)+P(C)-P\big((A\cup B)\cap C\big)$
$=\big(P(A)+P(B)-P(A\cap B)\big)+P(C)-P(A\cup B)\times P(C)$
$=P(A)+P(B)-P(A\cap B)+P(C)-\big(P(A)+P(B)-P(A\cap B)\big)\times P(C)$
$=x+y-xy+z-(x+y-xy)z$
$=x+y-xy+z-xz-yz+xyz$

iii. $P\big((A\cup B')\cap C\big)=P(A\cup B)\times P(C)$
$=\big(P(A)+P(B)-P(A\cap B)\big)\times P(C)$
$=(x+y-xy)z$
$=xz+yz-xyz$

4.3 With Conditional - Bayes Theorem

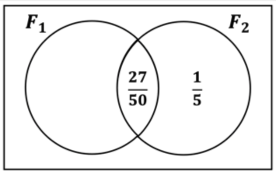
51)

Way 1: Without Using Bayes Theorem

$P(F_1) = \frac{3}{5} \Longrightarrow P(F_1') = \frac{2}{5}$

$$P(F_2|F_1) = \frac{9}{10}$$
$$\frac{P(F_1 \cap F_2)}{P(F_1)} = \frac{9}{10}$$
$$\frac{P(F_1 \cap F_2)}{\frac{3}{5}} = \frac{9}{10}$$
$$P(F_1 \cap F_2) = \frac{9}{10} \times \frac{3}{5} = \frac{27}{50}$$

$$P(F_2|F_1') = \frac{1}{2}$$
$$\frac{P(F_2 \cap F_1')}{P(F_1')} = \frac{1}{2}$$
$$\frac{P(F_2 \cap F_1')}{\frac{2}{5}} = \frac{1}{2}$$
$$P(F_2 \cap F_1') = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$$



$$P(F_1|F_2) = \frac{P(F_1 \cap F_2)}{P(F_2)}$$
$$= \frac{\frac{27}{50}}{\frac{27}{50} + \frac{1}{5}}$$
$$= \frac{27}{37}$$

Way 2: Bayes Theorem Formula

$$P(F_1|F_2) = \frac{P(F_1)P(F_2|F_1)}{P(F_1)P(F_2|F_1) + P(F_1')P(F_2|F_1')}$$
$$= \frac{\left(\frac{3}{5}\right)\left(\frac{9}{10}\right)}{\left(\frac{3}{5}\right)\left(\frac{9}{10}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{2}\right)}$$
$$= \frac{27}{37}$$

52)

Label events:
A = UN air flight
B = IS Air flight
L = luggage lost

We know; $P(A) = \frac{70}{70+65} = \frac{70}{135}$ and $P(B) = \frac{65}{135}$ and $P(L|A) = 0.18$, $P(L|B) = 0.23$

Using Bayes' theorem; $P(B|L) = \frac{P(L|B)\times P(B)}{P(L|B)\times P(B) + P(L|A)\times P(A)} = \frac{0.23\times\frac{65}{135}}{0.23\times\frac{65}{135} + 0.18\times\frac{70}{135}} \approx 0.543$

53)

i. Let percentage population vaccinated = p, which makes percentage population not vaccinated = $1 - p$

$P(catching\ virus) = percentage\ population\ vaccinated \times 0.1 + percentage\ population\ not\ vaccinated \times 0.3$

So $0.22 = p \times 0.1 + (1 - p) \times 0.3$

$$0.22 = 0.1p + 0.3 - 0.3p$$
$$0.2p = 0.3 - 0.22 = 0.08$$
$$p = \frac{0.08}{0.2} = 0.4 = 40\%$$

So, percentage of the population vaccinated = 40%

ii. Let events be:

A = person is vaccinated
B = person is not vaccinated
X = person catches virus

We know $P(A) = 0.4$ and $P(B) = 1 - 0.4 = 0.6$
$P(X|A) = 0.1, P(X|B) = 0.3$

Bayes theorem;

$$P(A|X) = \frac{P(X|A) \times P(A)}{P(X|A) \times P(A) + P(X|B) \times P(B)} = \frac{0.1 \times 0.4}{0.1 \times 0.4 + 0.3 \times 0.6} = \frac{0.04}{0.22} = \frac{2}{11}$$

54)

Label events:
A = Brian Air flight
B = Easy Flights flight
L = luggage lost

There are 3 times as many Brian Air flights as Easy Flights flights so $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$

We also know: $P(L|A) = \frac{1}{6}$ and $P(L|B) = \frac{1}{8}$

Using Bayes' theorem;

$$P(A|L) = \frac{P(L|A) \times P(A)}{P(L|A) \times P(A) + P(L|B) \times P(B)} = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{1}{8} \times \frac{1}{4}} = \frac{\frac{1}{8}}{\frac{5}{32}} = \frac{4}{5}$$

55)

Start by labelling events:
A = computer is from Factory A
B = computer is from Factory B
F = computer is faulty

Factory A produces double the number of batteries than Factory B so $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{3}$

We know: $P(F|A) = 0.15, P(F|B) = 0.2$

Using Bayes' theorem; $P(A|F) = \frac{P(F|A)\times P(A)}{P(F|A)\times P(A)+P(F|B)\times P(B)} = \frac{0.15\times\frac{2}{3}}{0.15\times\frac{2}{3}+0.2\times\frac{1}{3}} = \frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{15}} = \frac{3}{5}$

56)

$$P(F) = 0.9$$
$$P(M) = 0.1$$
$$P(R|M) = 0.95$$
$$P(R|F) = 0.08$$

Use Bayes' theorem:

$$P(M|R) = \frac{P(R|M)\times P(M)}{P(R|M)\times P(M) + P(R|F)\times P(F)} = \frac{0.95\times 0.1}{0.95\times 0.1 + 0.08\times 0.9} = \frac{95}{167} \approx 0.57$$

57)

Label events:

$$\text{P} = \text{mammogram result is positive}$$
$$\text{B} = \text{tumor is benign}$$
$$\text{M} = \text{tumor is malignant}$$

We know: $P(M) = 0.01$, $P(B) = 0.99$ and $P(P|M) = 0.8$, $P(P|B) = 0.1$

Bayes' theorem:

$$P(M|P) = \frac{P(P|M)\times P(M)}{P(P|M)\times P(M) + P(P|B)\times P(B)} = \frac{0.8\times 0.01}{0.8\times 0.01 + 0.1\times 0.99} = \frac{0.008}{0.107} \approx 0.075 = 7.5\%$$

7.5% is very far away from 75% so, **no, do not agree.**